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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. – Information Science and Engineering

Semester End Examination; Dec. - 2014

Discrete mathematical Structure

Time: 3 hrs *Max. Marks: 100*

***Note** : i) Answer **FIVE** full questions, selecting **ONE** full question from each Unit.*
ii) Assume suitable missing data if any.

Unit - I

1 a. Using Venn diagram PT for any three sets A, B, C

$\overline{(A\cup B)\cap C}=\left(\bar{A}\cap\bar{B}\right)\cup\bar{C}$ 7

b. Define power set, and for any sets A, B, C, D prove by using the laws of set theory that,

$(A\cap B)\cup\left(A\cap B\cap\bar{C}\cap D\right)\cup\left(\bar{A}\cap B\right)=B$ 6

c. Determine the co-efficient of :

i) x^9y^3 in the expression $(2x-3y)^{12}$ 7

ii) xyz^2 in the expression $(2x-y-z)^4$

2 a. In a sample of 100 logic chips, 23 have a defect D_1, 26 have a defect D_2, 30 have a defect D_3, 7 have defects D_1 and D_2. 8 have defects D_1 and D_3, 10 have defects D_2 and D_3, and 3 have all the 3 defects. Find the number of chips having (i) atleast one defect ii) no defect. 6

b. A problem given to four students A, B, C, D whole chances of solving it are $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}$ respectively, find the probability that the problem is solved. 6

c. Find how many distinct four digit integers one can make from the digits 1, 3, 3, 7, 7, 8. 8

Unit - II

3 a. Prove the validity of the given arguments using role of inference, 7

$$\begin{array}{l}(\neg pV\neg q)\rightarrow(r\wedge s)\\ r\rightarrow t\\ \neg t\\ \hline \therefore p\end{array}$$

b. Give i) a direct proof ii) an indirect proof for the given statement “If n is an odd integer : then n + 9 is an even integer” 6

c. Prove that for any three propositions p, q, r {using truth table} $\{(p\rightarrow q)\wedge(q\rightarrow r)\}\rightarrow(p\rightarrow r)$ is a tautology. 7

Contd…2

4 a. Find whether the following argument is valid: 8

No engineering student of first or second semester studies logic

<u>Anil is an Engineering student who studies logic</u>

$\therefore$ Anil is not in second semester.

b. Prove the logical equivalences using laws of logic, 7

$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$

c. Define quantified statements, and its truth valves. 5

Unit - III

5 a. Prove that for each n $\in Z^+$ 8

$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ using mathematical induction.

b. A sequence $\{an\}$ is defined recursively by $a_1 = 4, a_n = a_{n-1} + n$ for $n \geq 2$. Find an in explicit form. 7

c. Explain different types of functions. 5

6 a. Let $f : R \to R$ be defined by 8

$$f(x) = \begin{cases} 3x - 5 \text{ for } x > 0 \\ -3x + 1 \text{ for } x \leq 0 \end{cases}$$

Determine;

$f(0), f(-1), f\left(\frac{5}{3}\right)$

$f^{-1}(1), f^{-1}(-3), f^{-1}(-6)$

$f^{-1}([-5,5])$

b. Evaluate : $S(5,4)$ & $S(8,6)$ 6

c. Prove that in any set of 29 persons, atleast five persons must have born on the same day of the week. 6

Unit - IV

7 a. Let R be a relation defined as $a + b =$ even iff $(a,b) \in R$ on 8

$A = \{1, 2, 3, 5, 6, 10\}$

i) Write the relation matrix of R.

ii) Prove that R is an equivalence relation

iii) Draw the digraph of the relation.

b. Draw the Hasse diagram for give sets based on divisibility condition. 6

$i) A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$ii) B = \{2, 4, 8, 16, 32\}$

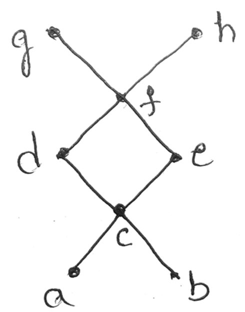
Contd…3

c. Define partial order, total order and equivalence relation with example each. 6

8 a. Let A = {a, b, c, d, e} consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. 6

b. Draw the Hasse diagram representing the positive divisors of 45. 7

c. Define Lattice, LUB, GLB of a poset, consider the Hasse diagram of a poset (A, R) given below. 7



If B = {c, d, e} find (if they exists)

i) all upper bound of B

ii) all lower bound of B

Unit - V

9 a. Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2}ab$. Show that (G, *) is an Abelian group. 8

b. The word C = 1010110 is sent through a binary symmetric channel. If P = 0.02 is the probability of incorrect receipt of a signal. Find the probability that 'C' is received as r =1011111. Determine the error pattern. 8

c. Define subgroup, cyclic group. 4

10 a. The generator matrix for an encoding function $E : Z_2^3 \to Z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

10

i) find the code words assigned to 110 and 010.

ii) Obtain the associated parity – check matrix.

iii) hence decode the received words : 110110, 111101

b. Define homomorphism and Isomorphism between two groups G_1 to G_2 6

c. Write short notes on Encoding and Decoding functions. 4

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