U.S.N ☐☐☐☐☐☐☐☐☐☐

P.E.S. College of Engineering, Mandya - 571 401
(An Autonomous Institution affiliated to VTU, Belgaum)
Third Semester, B.E. – Information Science and Engineering
Semester End Examination; Dec. - 2014
Discrete mathematical Structure

Time: 3 hrs   *Max. Marks: 100*

Note : i) Answer **FIVE** full questions, selecting **ONE** full question from each Unit.
  ii) Assume suitable missing data if any.

Unit - I

1 a. Using Venn diagram PT for any three sets A, B, C

$$\overline{(A \cup B) \cap C} = (\bar{A} \cap \bar{B}) \cup \bar{C}$$ 7

  b. Define power set, and for any sets A, B, C, D prove by using the laws of set theory that,

$$(A \cap B) \cup (A \cap B \cap \bar{C} \cap D) \cup (\bar{A} \cap B) = B$$ 6

  c. Determine the co-efficient of :

  i) $x^9 y^3$ in the expression $(2x - 3y)^{12}$ 7

  ii) xyz^2 in the expression $(2x - y - z)^4$

2 a. In a sample of 100 logic chips, 23 have a defect D_1, 26 have a defect D_2, 30 have a defect D_3, 7 have defects D_1 and D_2. 8 have defects D_1 and D_3, 10 have defects D_2 and D_3, and 3 have all the 3 defects. Find the number of chips having (i) atleast one defect ii) no defect.   6

  b. A problem given to four students A, B, C, D whole chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively, find the probability that the problem is solved.   6

  c. Find how many distinct four digit integers one can make from the digits 1, 3, 3, 7, 7, 8.   8

Unit - II

3 a. Prove the validity of the given arguments using role of inference,

$$(\neg p \lor \neg q) \to (r \land s)$$
$$r \to t$$
$$\underline{\neg t}$$
$$\therefore p$$ 7

  b. Give i) a direct proof   ii) an indirect proof for the given statement "If n is an odd integer : then n + 9 is an even integer"   6

  c. Prove   that   for   any   three   propositions   p,   q,   r   {using   truth   table}
  $\{(p \to q) \land (q \to r)\} \to (p \to r)$ is a tautology.   7

4 a. Find whether the following argument is valid:

No engineering student of first or second semester studies logic

Anil is an Engineering student who studies logic

∴ Anil is not in second semester.

8

b. Prove the logical equivalences using laws of logic,

$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$

7

c. Define quantified statements, and its truth valves.

5

Unit - III

5 a. Prove that for each $n \in Z^+$

$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ using mathematical induction.

8

b. A sequence $\{an\}$ is defined recursively by $a_1 = 4, a_n = a_{n-1} + n$ for $n \geq 2$. Find an in explicit form.

7

c. Explain different types of functions.

5

6 a. Let $f : R \to R$ be defined by

$f(x) = \begin{cases} 3x - 5 \text{ for } x > 0 \\ -3x + 1 \text{ for } x \leq 0 \end{cases}$

Determine;

$f(0), f(-1), f\left(\frac{5}{3}\right)$

$f^{-1}(1), f^{-1}(-3), f^{-1}(-6)$

$f^{-1}([-5,5])$

8

b. Evaluate : $S(5,4)$ & $S(8,6)$

6

c. Prove that in any set of 29 persons, atleast five persons must have born on the same day of the week.

6

Unit - IV

7 a. Let R be a relation defined as $a + b =$ even iff $(a,b) \in R$ on

$A = \{1, 2, 3, 5, 6, 10\}$

i) Write the relation matrix of R.

ii) Prove that R is an equivalence relation

iii) Draw the digraph of the relation.

8

b. Draw the Hasse diagram for give sets based on divisibility condition.

$i) A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$ii) B = \{2, 4, 8, 16, 32\}$

6

   c.   Define partial order, total order and equivalence relation with example each.   6

8 a.   Let A = {a, b, c, d, e} consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the   6
      equivalence relation inducing this partition.

   b.   Draw the Hasse diagram representing the positive divisors of 45.   7

   c.   Define Lattice, LUB, GLB of a poset, consider the Hasse diagram of a poset (A, R) given
      below.

      7
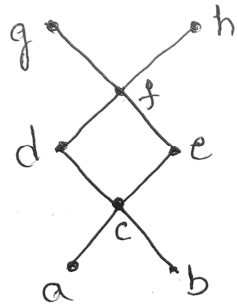
      If B = {c, d, e} find (if they exists)

      i) all upper bound of B

      ii) all lower bound of B

Unit - V

9 a.   Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2} ab$. Show that (G, *) is an   8
      Abelian group.

   b.   The word C = 1010110 is sent through a binary symmetric channel. If P = 0.02 is the
      probability of incorrect receipt of a signal. Find the probability that 'C' is received as   8
      r =1011111. Determine the error pattern.

   c.   Define subgroup, cyclic group.   4

10 a. The generator matrix for an encoding function $E : Z_2^3 \to Z_2^6$ is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

      10

      i) find the code words assigned to 110 and 010.

      ii) Obtain the associated parity – check matrix.

      iii) hence decode the received words : 110110, 111101

   b.   Define homomorphism and Isomorphism between two groups G_1 to G_2 6

   c.   Write short notes on Encoding and Decoding functions.   4

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