U.S.N [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

P.E.S. College of Engineering, Mandya - 571 401
(An Autonomous Institution affiliated to VTU, Belgaum)
Third Semester, B.E. - Information Science and Engineering
Semester End Examination; Dec.-2015
Discrete Mathematical Structure

Time: 3 hrs   *Max. Marks: 100*

Note: Answer any **FIVE** full questions, selecting **ONE** full question from each **unit**.

UNIT - I

1 a. How many arrangements are there for all letters in the word SOCIOLOGICAL

   (i) How many of these arrangements 'A' and 'G' are adjacent   7

   (ii) How many of these arrangements all the vowels are adjacent?.

 b. For any sets A, B, C, D prove by this using the laws of set theory that

   $$(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B) = B$$ 7

 c. A problem is given to four students A, B, C, D whose chances of solving it are

   $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively find the probability that problem is solved.   6

2 a. Using Venn diagram, prove that for any 3 sets A, B, C $\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$ 7

 b. If A,B,C are finite sets prove the following extended addition principle

   $$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$ 6

 c. A women has 11 close relatives and she wishes to invite 5 of them to dinner in how many

   ways can she invite them in the following situations She invite them in the following situation

   (i) There is no restriction on the choice   7

   (ii) Two particular persons will not attend separately

   (iii) Two particular persons will not attend together.

UNIT - II

3 a. Prove that, for any proportions p, q, r the compound proportion

   $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ is a tautology.   7

 b. Prove the following logical equivalence without using truth table:

   $$[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$$ 6

 c. Test whether the following argument is valid

   $p \rightarrow q$

   $r \rightarrow s$

   $\underline{p \vee r}$ 7

   $\therefore q \vee s$

4 a. Prove the validity of the following argument:

$$\left[(\neg p \vee \neg q) \rightarrow (r \vee s)\right] \wedge \left[r \rightarrow t\right] \wedge \left[\neg t\right] \Rightarrow p$$

7

b. Find whether the following argument is invalid

No Engineering student of 1st or 2nd semester studies logic

Anil is an engineering student who studies logic

∴ Anil is not in Second Semester.

7

c. Let p, q, r be propositions having truth valves F, F, T respectively. Find the truth values of the following compound propositions:

i) $(p \vee q) \vee r$ ii) $(p \wedge q) \wedge r$ iii) $(p \wedge q) \rightarrow r$

iv) $p \rightarrow (q \wedge r)$ v) $P \wedge (r \rightarrow q)$ vi) $p \rightarrow (q \rightarrow (\neg r))$

6

UNIT - III

5 a. Give a (i) direct proof (ii) An Indirect proof

(iii) Proof by contradiction, for the following statement.

" If n is an odd integer; then n + 9 is an even integer"

7

b. Prove by mathematical induction that for all positive integers

$n \geq 1, \quad 1 + 2 + 3 + 4 + \dots\dots\text{n} = \frac{1}{2} n(n+1)$

6

c. Find the number of ways of distribution 4 distinct objects among three identical containers, with some containers possibly empty.

7

6. a. ABC is equilateral triangle whole sides are of length 1 cm each. If we select 5 points inside the triangle. Prove that at least two of these points are such that the distance between than is less than $\frac{1}{2}$ cm.

6

b. Let $f : R \rightarrow R$ be defined by

$$f(0) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ 3x + 1 & \text{for } x \leq 0 \end{cases}$$

Determine $f(0), f(-1), f\left(\frac{5}{3}\right), f^{-1}(1), f^{-1}(3), f^{-1}(-3)$

7

c. Let f, g, h be functions from R to R defined by $f(x) = x + 2, \ g(x) = x - 2, \ h(x) = 3x$ for all $x \in R$ Find $gof, \ fog, \ gog,$ hof, $hog, \ fof, \ fohog$

7

UNIT - IV

7 a. If $A = \{1, 2, 3, 4\}$ & R is a relation on A defined by $R = \{(1,2)(1,3)(2,4)(3,2)(3,3)(3,4)\}$

Find R^2 & R^3 Verify that $M(R^2) = \left[M(R)\right]^2$ & $M(R^3) = \left[M(R)\right]^3$

6

b. Find the number of equivalence relations that can be defined on finite Set A with $|A| = 6$ 6

c. Draw Hasse diagram representing the positive divisors of 36.   8

8 a. For the poset (A, R) defined on the set $A = \{2, 3, 5, 6, 7, 11, 12, 35, 385$ as represented by the following Hasse diagram find

$(L \cup B)\{2, 3\}, \quad (L \cup B)\{2, 12\}, \quad (L \cup B)\{3, 6\}, \quad (L \cup B)\{5, 7\}$

$GLB\{2, 12\}, \quad GLB\{3, 6\}, \quad GLB\{6, 12\}, \quad GLB\{35, 385\}$
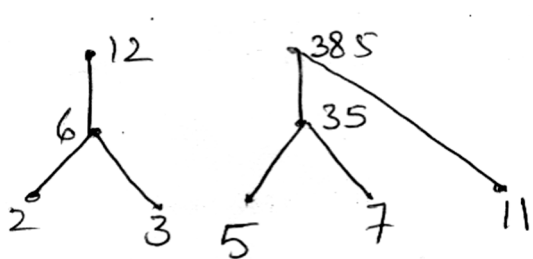
8

b. The digraph for a relation on set $A = \{1, 2, 3, 4\}$ is as shown below.
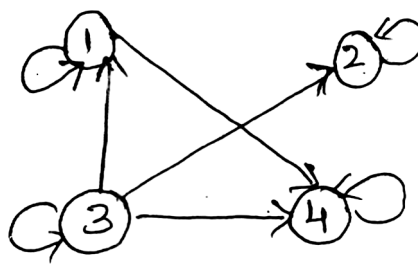
6

i) verify that (A, R) is a poset and Draw its Hasse diagram

ii) How many more directed edges are needed in the digraph of 'R' to extent 'R' to a total order?

c. Let $A = \{a, b, c, d, e\}$ consider the partition $P = \{\{a, b\}\{c, d\}\{e\}\}$ of A Find the equivalence relation inducing this partition   6

UNIT - V

9 a. Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2} ab$ Show that $(G, *)$ is an Abelian group.   7

b. State and prove Lagrange's theorem.   6

c. The word c = 1010110 is sent through a binary symmetric channel. If p = 0.02 is the probability that c is received as $r = 1011111$ Determine the error pattern.   7

10 a. For the group $G = (Z_{12} +)$ and the subgroup $H = \{[0], [4], [8]\}$ find all the left coset of H in G. Also, obtain the corresponding coset decomposition of G.   7

b. The parity check matrix for an encoding function $E : Z_2^3 \rightarrow Z_2^6$ is given by

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

6

i) Determine the associated generator matrix.

ii) Does this code correct all single errors in transmission?

c. Find all integers k and m for which $(z, \oplus, \Theta)$ is a ring under the binary operations

7

$x \oplus y = x + y - k, \quad x \Theta y = x + y - mxy$

✼ ✼ ✼ ✼