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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Information Science and Engineering

Semester End Examination; June/July - 2015

Graph theory and Combinatories

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each **Unit**.*

UNIT - I

1. a. Prove that "In every graph, the number of vertices of odd degree is even" 6

b. Determine the order $|V|$ of the graph G(V, E) in the following cases: 7

i) G is a cubic graph with 9 edges

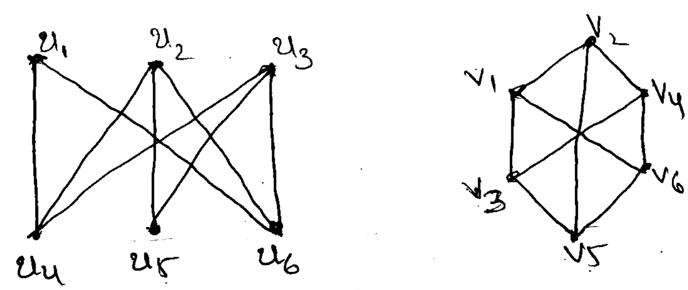
ii) G has 10 edges with 2 vertices of degree 4 and all the other vertices of degree 3

iii) G is regular with 15 edges.

c. Define Handshaking property. For a graph with n vertices and 'm' edges if '8' is the minimum and 'Δ' is the maximum of the degrees of vertices. Show that $8 \leq \frac{2m}{n} < \Delta$ 7

2 a. Show that every simple graph of order ≥ 2 must have at least two vertices of the same degree. 6

b. Define Isomorphism of a graph. Verify the given graphs are Isomorphic or not. 8

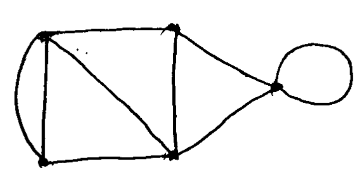


c. Define sub graph, Bipartite graph, complete graph, Regular graph, Euler circuit and Euler trail. 6

UNIT - II

3 a. Define a planar graph & prove that K_5 is non-planar graph. 7

b. Carryout the elementary reduction process for the following graph to detect planarity. 6

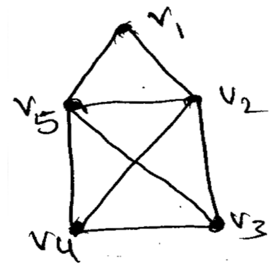


c. State Euler formula. A connected graph G has 9 vertices with degrees 2, 2, 3, 3, 3, 4, 5, 6, 6. Find the number of regions of G. 7

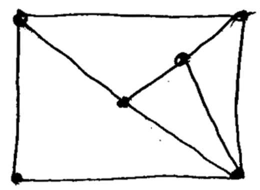
Contd…2

4 a. Write the steps involved in Elementary Reduction" method to detect planarity. 6

b. Using multiplication theorem, find the chromatic polynomial for the graph below. If 5 colours are available, in how many ways can be vertices of this graph be property coloured? 7



c. Define platonic solids? Homeomorphic graphs? Write the dual the given graph. 7



UNIT - III

5 a. Define a tree. Prove the following theorems :

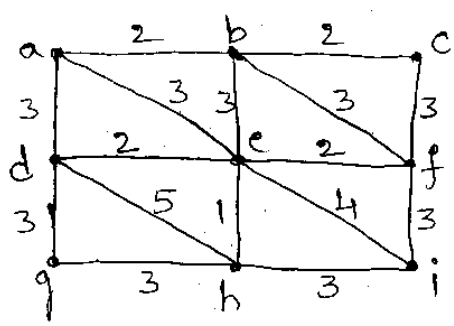
i) In a tree, there is one and only one only path between every pair of vertices 7

ii) A connected graph is a tree. iff it is minimally connected.

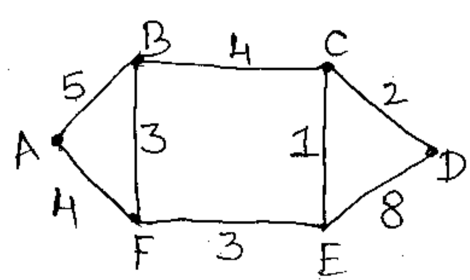
b. Suppose that a tree 'T' has two vertices of degree 2. Four vertices of degree 3 and 3 vertices of degree 4. Find the number of pendant vertices in T. 6

c. Obtain an optimal prefix code for the message "ROAD IS GOOD". Indicate the code. 7

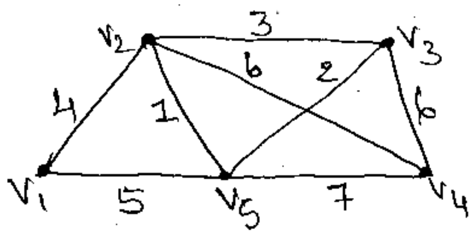
6. a. Using Kurskal's method, determine a minimal Spanning tree for the graph shown below: 7



b. Define Cut-sets. For the networks shown below: determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. 6



c. Using Prim's Algorithm find a minimal spanning tree for the weighted graph shown below: 7

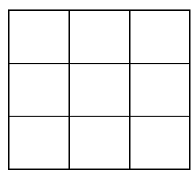


UNIT - IV

7 a. Determine the number of positive integers 'n' such that $1 \leq n \leq 100$ and 'n' is not divisible by 2, 3, 5. 7

b. Define derangements. There are eight letters to eight different people to be placed in eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this. So that atleast one letter gets to the right person. 6

c. Find the rook polynomial for 3 x 3 board shown below: 7



8.a. Find the generating functions for the following sequences: 6

i) $1^2, 2^2, 3^2, 4^2, \ldots\ldots.$

ii) $1^3, 2^3, 3^3, 4^3, \ldots\ldots\ldots$

b. There are 10 marbles of the same size but of different colours (1 red, 3 blue, 3 green, 2 orange, 1 white and 1 black) in a bag. Find in how many ways six of these are marbles can be arranged in a row. 7

c. Find how many distinct 4-digit and 5-digit integers one can make from the digits 1, 3, 3, 7, 7, 8. 7

UNIT - V

9 a. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$ 7

b. Solve the recurrence relation: $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$. Given that $a_0 = 1, a_1 = 2$ 7

c. Solve the recurrence relation: $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ given $a_0 = 1$, $a_1 = 4$ and $a_2 = 28$ 6

10.a. Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$ for $n \geq 0$ with given $a_0 = 0, a_1 = 1$ and $a_2 = 2$. 7

b. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$ given that $a_0 = -1$, and $a_1 = 8$ 7

c. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42…. Hence find the general term of the sequence. 6

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