



P.E.S. College of Engineering, Mandya - 571 401
(An Autonomous Institution affiliated to VTU, Belgaum)
Fourth Semester, B.E. - Information Science and Engineering
Semester End Examination; June - 2016
Graph theory and Combinatorics

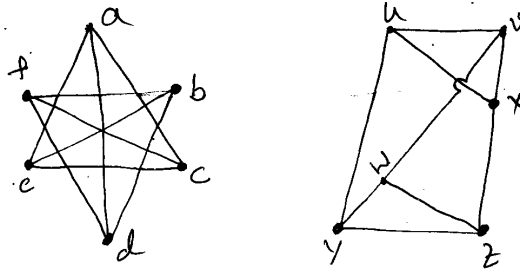
Time: 3 hrs

Max. Marks: 100

Note: Answer FIVE full questions, selecting ONE full question from each unit.

UNIT - I

1. a. What is Graph Isomorphism? Determine whether or not the given 2 graphs Isomorphic.



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b. Seven towns a, b, c, d, e, f and g are connected by a system of Highways,

- I) I - 22 goes from a to c passing through b,
- II) I - 33 goes from c to d passing through b as it continues to f
- III) I - 44 goes from d through e to a
- IV) I - 55 goes from f to b, passing through g and
- V) I - 66 goes g to d.

- i) Using vertices for towns and directed edges for segments of highway between towns, draw a directed graph that model this situation.
- ii) List the paths from g to a.
- iii) What is the smallest number of highway segments that would have to be closed down in order for travel from b to d to be disrupted?
- iv) Is it possible to leave town C and return there, visiting each other towns only once?
- v) What is the answer to part (iv) if we are not required to return C?
- vi) Is it possible to start at some town and drive over each of these highways exactly once?

12

2 a. Determine $|V|$ for the following graphs or multigraphs G,

- i) G has nine edges and all vertices have degree 3
- ii) G is a Regular with 15 edges
- iii) G has 10 edges with 2 vertices of degree 4 and all others of degree 3.

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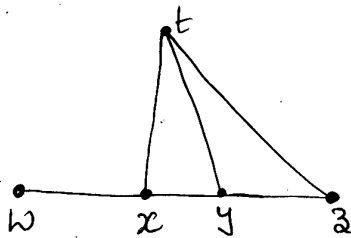
b. With suitable example explain the following :

- i) Complement of a graph
- ii) Euler trail and Euler circuit
- iii) Hamilton path and Hamilton cycle
- iv) Subgraph and a Spanning subgraph
- v) Regular graph.

14

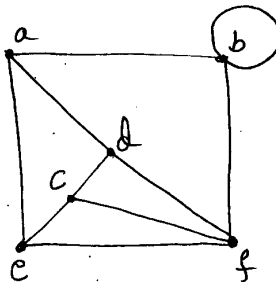
UNIT - II

3 a. What is chromatic number and chromatic polynomial of a graph? Find the chromatic polynomial of the given graph. If 25 colors are available, in how many ways the graph can be colored?



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b. Find the dual of a given graph. Explain each step.



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4 a. Prove that a graph is non planar iff it contains a subgraph that is homeomorphic to either K_5 or $K_{3,3}$.

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b. Prove that $v - e + r = 2$ with usual notations.

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c. Find the maximum length of a trail in K_6 .

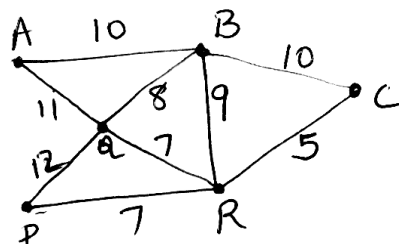
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UNIT - III

5 a. Generate the optimal prefix code for the message "SPEED THRILLS BUT KILLS". Using labeled binary tree. Indicate the code.

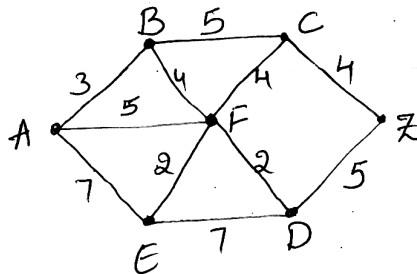
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b. Using Kruskal's algorithm. Find a minimal spanning tree for the given weighted graph.

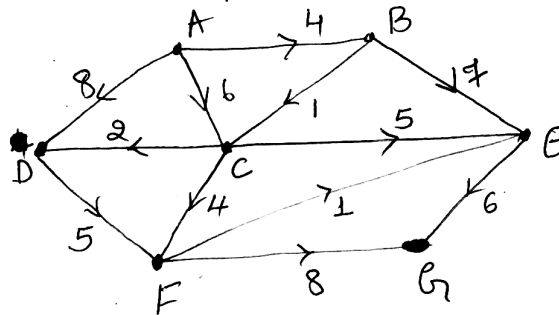


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- c. Prove in every tree $T(V, E)$, $|V| = |E| + 1$. 6
- 6. a. Explain the steps involved in Prim's algorithm by considering suitable example. 6
- b. Find the possible maximum flow between A and Z in the given network.



- c. Using Dijkstra's algorithm obtain the shortest path from vertex A to each of the other vertices in the weighted directed Network. 6



UNIT - IV

- 7 a. Determine the number of positive integers n , where $1 \leq n \leq 200$ that are not divisible by 2, 3, 5 or 7. 12
- b. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to the right person. 8
- 8.a. Five teachers T_1, T_2, T_3 and T_4 and T_5 are to be made class teachers for five class C_1, C_2, C_3, C_4, C_5 , one teacher for each class. T_1 and T_2 do not wish to become class teachers for C_1 or C_2 . T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5 . In how many ways can the teachers be assigned the work, without displeasing any teacher? 10
- b. Find the generating function for the following sequence :
 - i) 1, 1, 0, 1, 1, 6
 - ii) 0, 2, 6, 12, 20, 30,
 - iii) 8, 26, 54, 92,
- c. Using the exponential generating function, find the number of ways in which four of the letters in the word "HAWAII" be arranged. 4

UNIT - V

- 9 a. Find a recurrence relation with initial condition for the geometric progression 2, 10, 50, 250, 5
- b. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 7^n$ for $n \geq 1$, given $a_0 = 2$. 10
- c. The number of viruses affected files in a system is 1000 and this increases 250% every 2 hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. 5
- 10.a. Find the generating function for the recurrence relation, 12
- $$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2, r \geq 2 \text{ and } a_0 = a_1 = 0.$$
- b. Suppose there are $n \geq 2$ persons at party and that each of these persons shakes hands exactly once with all of the other persons present. Using recurrence relation, find the number of handshakes. 8

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