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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester - Master of Computer Applications (MCA)

Semester End Examination; Jan/Feb. - 2016

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. Find the number of permutations of the letters of the word "MASSASAUGA". In how many ways of these all four A's are together? How many of them beginning with S? 6

b. Find the value of n, so that 2P (n, 2) +50 = P (2n, 2). 6

c. Prove the following identities : 8

(i) C(n+1, r) = C(n, r-1) + C(n, r) (ii) C(m+n, 2) - C(m, 2) - C(n, 2) = mn

2 a. Using laws of set theory show that : 5

$$(A \cap B) \cup \left[(A \cap B) \cap (\overline{C} \cap D) \right] \cup (\overline{A} \cap B) = B$$

b. Using Venn diagram P.T. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$ for any three sets A, B, C. 5

c. 40 computer programmers interviewed for a job. 25 knew JAVA, 28 knew ORACLE and 7 knew neither language. How many knew both the languages. 5

d. A problem is given to 4 students. A, B, C, D whose chances of solving it are $1/2,\ 1/3,\ 1/4,\ 1/5$ respectively. Find the probability that the problem is solved. 5

UNIT - II

3 a. Define tautology. Without constructing the truth table prove that : 7

$$\{(p \vee q) \wedge \sim [\sim P \wedge (\sim q \vee \sim r)]\} \vee \{(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)\}$$ is a tautology.

b. By constructing the truth table, prove that $[p \leftrightarrow (q \leftrightarrow r)] \equiv [(p \leftrightarrow q) \leftrightarrow r]$. 6

c. Prove that the given argument is valid, 7

$$\begin{array}{l} p \rightarrow (q \wedge r) \\ r \rightarrow s \\ \underline{\sim (q \wedge s)} \\ \therefore \sim P \end{array}$$

4 a. Explain Direct proof, Indirect proof and contradiction proof. 7

b. Consider the following open statements with the set of all real numbers as universe: 6

$$p(x):\ x \geq 0,\quad q(x): x^2 \geq 0,\quad r(x): x^2 - 3x - 4 = 0,\quad s(x) = x^2 - 370$$

Contd…..2

Determine the truth values of the following quantified statements :

i) $\exists x,\ p(x) \wedge q(x)$ ii) $\forall x,\ p(x) \rightarrow q(x)$

iii) $\forall x,\ q(x) \rightarrow s(x)$ iv) $\forall x,\ r(x) \vee s(x)$ v) $\exists x,\ p(x) \wedge r(x)$

c. Prove that the given argument is valid

If Δ^{le} has 2 equal sides than it is isosceles

If Δ^{le} is isosceles than it has 2 equal angles 7

The Δ^{le} ABC does not have 2 equal angles.

$\therefore$ ABC does not have 2 equal angles.

UNIT - III

5 a. Using mathematical induction. Prove that, $1+3+5+\ldots\ldots+(2n-1) = n^2 \ \forall\ n \geq 1$ 6

b. The Lucas numbers are defined recursively by $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1}+L_{n-2}$ for $n \geq 2$. Evaluate L_2 to L_{10} and also state induction principle. 6

c. Let $|A| = m$ *and* $|B| = n$. Find how any functions are possible from A to B. If there are 2187 functions from A to B and $|B| = 3$ then what is $|A|$? 8

6 a. State stirling number of second kind and verify that S (5, 3) = 25, S(7, 2) = 63. 7

b. Prove that in any set of 29 persons atleast 5 persons must have been born on the same day of the week. 7

c. Consider the functions f & g defined by $f(x) = x^3$ and $g(x) = x^2+1$, $\forall x \in R$. Find fog, gof, f^2 and g^2. 6

UNIT - IV

7 a. Define POSET and construct the Hasse diagram of all + ve divisors of 36. 7

b. If A = {1, 2, 3, 4, 5, 6} and R is a relation on A defined by R = {(1, 2), (1, 6), (2, 3), (3, 3) , (3,4), (4, 1), (4, 3), (4, 5), (6, 4)}

(i) List all the paths of length 3 originating at the vertex 1. 7

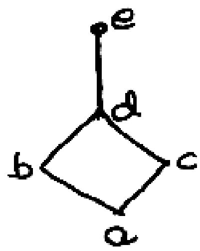
(ii) List all the cycles of length 4

(iii) List the length of path from 6 to 2.

c. If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1, 2\}$, $A_2 = \{2, 3, 4\}$ and $A_3 = \{5\}$. Define the relation R on A by xRy iff x and y are in the same set A_i, i = 1, 2, 3. Is R satisfies an equivalence relation. 6

8 a. On the set Z of all integers, a relation on R is defined by aRb if $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced. 7

b. For A = {a, b, c, d, e} the Hasse diagram for the POSET (A, R) is as shown below : 8



(i) Determine the relation matrix for R

(ii) Construct the di-graph for R

(iii) Topologically sort the POSET (A,R)

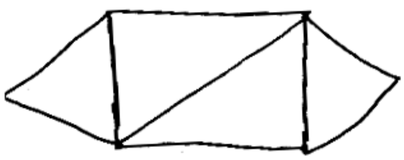
c. Define : Maximal element, Minimal element Infimum and Supremum. 5

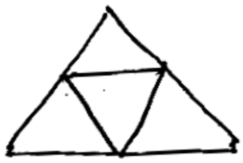
UNIT - V

9 a. Define: 6

(i) Degree of a vertex (ii) Regular graph (iii) Complete graph with example each.

b. Define Isomorphism. Verify whether the following graphs are isomorphic (or) not and justify. 7





c. Define tree. For every tree T = (V, E). If $|V| \geq 2$, Show that, T has atleast 2 pendant Vertices. 7

10 a. Prove that a graph of order $n(\geq 2)$ consisting of a single cycle is 2-chromatic if n is even and 3-chromatic if n is odd. 6

b. Construct an optimal prefix code for the frequencies {4, 15, 25, 5, 8, 16} 7

c. Define minimal spanning tree. Write an algorithm to find minimal spanning tree using Prim's algorithm. 7

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