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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, Master of Computer Applications (MCA)

Semester End Examination; Jan - 2017

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them, if he has 3 servants to carry the cards? 6

b. State Binomial theorem and Multinomial theorem. Find the coefficient of $a^2b^3c^2d^5$ in the expansion $(a + 2b - 3c + 2d + 5)^{16}$. 7

c. Find the number positive integer solutions of the equation,

$x_1 + x_2 + x_3 + x_4 = 25 \quad \forall\, x_i \geq 0.$ 7

2 a. Define symmetric differences of sets. For any 2 set A and B, simplify: $\overline{\left((A \cup B) \cap C\right) \cup \overline{B}}$. 6

b. Show that $\overline{A \cup (B \cap C)} = \overline{A} \cap (\overline{B} \cup \overline{C})$ by membership method. 7

c. Among the first 500 positive integers,

i) Determine the integers which are not divisible by 2 nor by 3, nor by 5. 7

ii) Determine the integers which are exactly divisible by one of them.

UNIT - II

3 a. Define logical equivalence. By constructing the truth table, prove that $\{[P \wedge (\neg P \vee q)] \vee [q \wedge \neg(p \wedge q)]\} \equiv q.$ 6

b. By using laws of logic theory, prove that $(p \leftrightarrow q) \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$ is tautology. 7

c. State rule of disjunctive syllogism and rule of disjunctive amplification. Test whether the given argument is valid (or) not? 7

$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ \quad p \\ \hline \therefore\; r \end{array}$$

4 a. Write down the following propositions in symbolic form and also find their negations,

i) For all integers '*n*', If '*n*' is not divisible by 2, then *n* is odd 6

ii) If all triangles are right angled then no triangle is equiangular.

b. Prove that the given argument is valid, 7

$$\begin{array}{l} \forall x, \quad p(x)q(x) \\ \neg p(x) \\ \neg q(x) \vee r(x) \\ \underline{s(x) \rightarrow \neg r(x)} \\ \therefore \neg s(a) \end{array}$$

c. Explain direct proof. Give a direct proof for the given statement, "Square of an odd integer is an odd integer". 7

UNIT - III

5 a. Prove that, $4n \le n^2 - 7, \ \forall$ integers i.e. $n \ge 6$ using mathematical induction. 6

b. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \ge 2$. Evaluate F_2 to F_{10}. 7

c. Show that the closed binary operation f: R x R $\rightarrow$ R defined by f(a, b) = $\lceil a + b \rceil$ is commutative but not associative. 7

6 a. Define :

i) One-one function ii) Onto function iii) Identity function 6

iv) Constant function v) Floor function vi) Ceiling function with example each.

b. Let f : R $\rightarrow$ R be defined by, $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$ 7

Determine, $f(0), f\left(-5/3\right), f^{-1}(1), f^{-1}(-6), f^{-1}(3), f^{-1}([-5,5]), f^{-1}([-6,5])$.

c. State Pigeonhole principle. Let *S* = {3, 7, …, 103}. How many elements must we select from *S* to insure that there will be at least two whose sum is 110? 7

UNIT - IV

7 a. If A is a set with *m* elements and B is a set with *n* elements. Find the number of relations from A to B: If there are 4096 relations from A to B and |B| = 3. Then what is |A| = ? 6

b. Let A = {1, 2, 3, 4, 6, 8, 12} on A define relation R by "aRb". Iff "a divides b". Prove that R is a POSET on A. Draw the Hasse diagram for this relation. 7

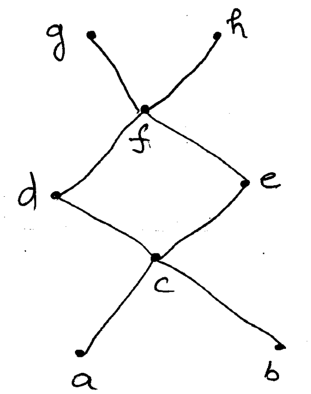
c. Let R = {(1, 2), (3, 4), (2, 2)} and S = {(4, 2), (2, 5), (3, 1), (1, 3)} be the relations on the set A = {1, 2, 3, 4, 5}. Find the following, 7

$R \circ (R \circ S), \ R \circ (S \circ R), \ S \circ (R \circ S), \ S \circ (S \circ R)$.

8 a. Let *f* and *g* be the functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $gof(x) = 9x^2 - 9x + 3$. Determine *a* and *b*. 6

b. Define: i) Maximal element ii) Supremum iii) Infimum iv) Lattice. 7

c. Consider the Hasse diagram of a poset (A, R) given below, 7



If B = {c, d, e} find;

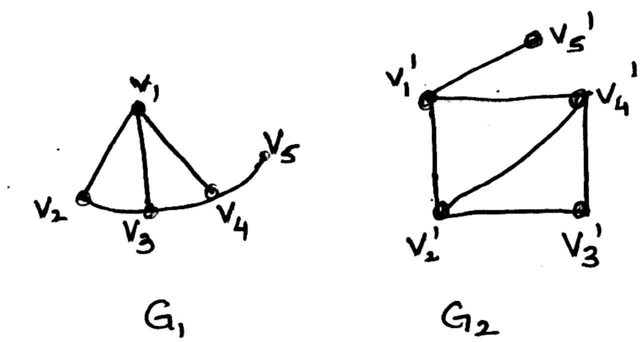
i) all upper bounds of B ii) all lower bounds of B

iii) least upper bound of B iv) greatest lower bound of B.

UNIT - V

9 a. Define simple graph. Prove that in any simple graph with *n* vertices, the number of edges is at most $\frac{n(n-1)}{2}$. 6

b. Define isomorphism. Prove that the graphs shown below are isomorphic, 7



c. i) Define Euler and Hamiltonian graphs with examples.

ii) Draw a graph which is both Euler and Hamiltonian. 7

iii) Draw a graph Hamiltonian but not Euler.

10 a. Define minimally connected graph. Prove that a graph G is minimally connected iff it is a tree. 6

b. Define planar and non planar graphs. How many edges must a planar graph must have, if it has 7 regions and 5 vertices? Draw one such graph. 7

c. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with the frequencies 20, 28, 4, 17, 12, 7 respectively. 7

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