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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, B.E. – Make-up Examination; Jan/Feb - 2016

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. Find the n[th] derivative of (i) $y=\log_{10}\sqrt{(2x+3)(4-3x)}$ (ii) $y=e^{2x}.\sin^3 x$ 6

b. Find the n[th] derivation of $y=\dfrac{x^2}{(2x-1)(x^2-6x+9)}$ 7

c. If $y=e^{m\sin^{-1}x}$, Prove that, 7
$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(m^2+n^2)y_n=0$

2 a. State Lagrange's mean value theorem and verify the same for the function: 6
$f(x)=(x-1)(x-2)(x-3)$ in [0, 4]

b. Obtain the Taylor's series expansion of $\log_e x$ in powers of (x – 1) upto sixth degree terms and hence find $\log_e(1.1)$ 7

c. Using Maclaurin's series. Expand $\log_e \sec x$ in ascending powers of x upto the term containing x^6. 7

UNIT - II

3 a. Evaluate : (i) $\underset{x\to 0}{lt}\left(\dfrac{\tan x-x}{x^2\tan x}\right)$ (ii) $\underset{x\to 0}{lt}\left(\log_{\sin x}\sin 2x\right)$ 6

b. Evaluate; (i) $\underset{x\to\frac{\pi}{2}}{lt}(\sec x-\tan x)$ (ii) $\underset{x\to a}{lt}\left(2-\dfrac{x}{a}\right)^{\tan\frac{\pi x}{2a}}$ 7

c. Find the angle of intersection of the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$ 7

4 a. Find the Pedal equation of the curve $r^m=a^m(\cos m\theta+\sin m\theta)$ 6

b. For the cardioid $r=a(1+\cos\theta)$, show that ρ^2/r is constant. 7

c. Show that the radius of curvature of the curve $x=a\cos^3\theta,\ y=a\sin^3\theta$ is $3a\sin\theta\cos\theta$ 7

UNIT - III

5 a. If $z(x+y)=x^2+y^2$, show that $\left(\dfrac{\partial z}{\partial x}-\dfrac{\partial z}{\partial y}\right)^2=4\left(1-\dfrac{\partial z}{\partial x}-\dfrac{\partial z}{\partial y}\right)$ 6

b. State Euler's theorem for a function of two variables and use it to find $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$, if $u=\sin^{-1}\left(\dfrac{x^2y^2}{x-y}\right)$ 7

c. If $u=f(x,y)$ and $x=r\cos\theta, y=r\sin\theta$. Prove that $\left(\dfrac{\partial u}{\partial x}\right)^2+\left(\dfrac{\partial u}{\partial y}\right)^2=\left(\dfrac{\partial u}{\partial r}\right)^2+\dfrac{1}{r^2}\left(\dfrac{\partial u}{\partial \theta}\right)^2$ 7

6 a. A particle moves along the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is time. Find the components of velocity and acceleration at t=1, in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$ 6

b. Find div $\vec{F}$ and curl $\vec{F}$ at the point (1, 2, 3), if $\vec{F}$ = grad $\left(x^3 + y^3 + z^3 - 3xyz\right)$ 7

c. Find a, b, c such that $\vec{F} = \left(x + y + az\right)\hat{z} + \left(bx + 2y - z\right)\hat{j} + \left(x + cy + 2z\right)\hat{k}$ is irrotational. Hence find ϕ such that $\vec{F} = \nabla\phi$ 7

UNIT - IV

7 a. Obtain a reduction formula for $\int \cos^n x\, dx$ and $\int_0^{\pi/2} \cos^n x dx$, where n is a positive integer. 6

b. Evaluate: i) $\int_0^{\pi} x \sin^8 x dx$ ii) $\int_0^{\infty} \frac{x^4}{\left(1 + x^2\right)^4} dx$. 7

c. Trace the curve: $y^2\left(a - x\right) = x^3,\ a > 0$ 7

8 a. Find the area enclosed by the Astroid: $x^{2/3} + y^{2/3} = a^{2/3}$ 6

b. Find the surface area of revolution of the curve $r = a\left(1 + \cos\theta\right)$ about the initial line. 7

c. By differentiating under the integral sign, evaluate $\int_0^{\infty} \frac{e^{-\alpha x} \sin x}{x} dx$ and hence evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$ 7

UNIT - V

9 a. Solve : $\left(x \tan\frac{y}{x} - y \sec^2\frac{y}{x}\right)dx + x \sec^2\left(\frac{y}{x}\right)dy = 0$ 6

b. Solve : $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ 7

c. Solve : $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$ 7

10a. Solve : $\left(x^2 + y^3 + 6x\right)dx + xy^2 dy = 0$ 6

b. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter. 7

c. When a resistance R omhs is connected in series with an inductance L henries, an e.m.f. E volts, the current i amperes at any time t is given by $L\frac{di}{dt} + Ri = E$. if E = 10 sint volts and i = 0, when t = 0. Find i as a function of t. 7

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