U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, B.E. - Semester End Examination; Dec. - 2015

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs Max. Marks: 100

***Note**: Answer **FIVE** full questions selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. Find the n[th] derivative of the following functions:
(i) $\log_{10}\left[(1-2x)^3(8x+1)^5\right]$ (ii) $e^x \cos x \cos 3x$ 6

b. Find the n[th] derivative of the function $y=\dfrac{x^2}{(2x+1)(2x+3)}$ 7

c. If $y^{1/m}+y^{-1/m}=2x$, then using Leibritz's theorem show that
$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$ 7

2 a. State Rolle's theorem, and verify Lagrange's mean value theorem for the function:
$f(x)=(x-1)(x-2)(x-3)$ in [0, 4] 6

b. Verify Cauchy's mean value for the functions $\log_e x$ and $\dfrac{1}{x}$ in [1, e] 7

c. Expand $\log_e\left(x+\sqrt{x^2+1}\right)$ by using Maclaurin's theorem upto and including the term containing x^3. 7

UNIT - II

3 a. Evaluate ; (i) $\underset{x\to 0}{lt}\dfrac{e^{2x}-(1+x)^2}{x\log_e(1+x)}$

(ii) $\underset{x\to\infty}{lt}\left(a^{1/x}-1\right)x$ 6

b. Evaluate; (i) $\underset{x\to 0}{lt}\left(\dfrac{1}{x}-\cot x\right)$

(ii) $\underset{x\to 0}{lt}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$ 7

c. Find the angle of intersection of the curves $r=\sin\theta+\cos\theta$ and $r=2\sin\theta$ 7

4 a. Find the Pedal equation of the curve $r^m=a^m(\cos m\theta+\sin m\theta)$ 6

b. Find the radius of curvature of the curve $x=a\left[\cos t+\log\tan\left(\dfrac{t}{2}\right)\right]: y=a\sin t$ at t. 7

c. Show that for the curve $r^n=a^n\cos n\theta$ the radius of curvature is $\dfrac{a^n}{(n+1)r^{n-1}}$ 7

UNIT - III

5 a. If $z(x+y)=x^2+y^2$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$ 6

b. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that $x^2u_{xx}+y^2u_{yy}+2xyu_{xy}=\sin 4u-\sin 2u$ 7

c. If $u=\sin^{-1}(x-y)$ where $x=3t, y=4t^3$ show that $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$ and verify the result by direct differentiation. 7

6 a. Find the components of velocity and acceleration at t = 2 on the curve $\vec{r}=(t^2+1)\hat{i}+(4t-3)\hat{j}+(2t^2-6t)\hat{k}$ in the direction $\hat{i}+2\hat{j}+2\hat{k}$ 6

b. Find the directional derivative of $\phi=\frac{xz}{x^2+y^2}$ at (1, -1, 1) in the direction of $\vec{A}=\hat{i}-2\hat{j}+\hat{k}$ 7

c. Show that $\vec{F}=(2xy^2+yz)\hat{i}+(2x^2y+xz+2yz^2)\hat{j}+(2y^2z+xy)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$ 7

UNIT - IV

7 a. Obtain a reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^{\pi/2}\sin^n x\,dx$ where n is a positive integer. 6

b. Evaluate: $\int_0^{2a} x^2\left(\sqrt{2ax-x^2}\right)dx$ 7

c. Trace the curve: $y^2(a-x)=x^3,\ a>0$ 7

8 a. Find the area bounded by an arch of the cycloid $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta),\ 0\le\theta<2\pi$ and its base. 6

b. Find the volume generated by the revolution of the curve $r=a(1+\cos\theta)$ about the initial line. 7

c. By using the rule of differentiation under the integral sign, evaluate; $\int_0^1 \frac{x^a-1}{\log x}dx$, where a is a parameter ≥ 0. 7

UNIT - V

9 a. Solve : $(y^3-3x^2y)dx-(x^3-3xy^2)dy=0$ 6

b. Solve : $\frac{dy}{dx}=\frac{x+y-1}{x-y+1}$ 7

c. Solve : $6y^2dx-x(x^3+2y)dy=0$ 7

10a Solve : $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$ 6

b. Find the orthogonal trajectories of the family $r^n\cos n\theta=a^n$. 7

c. If the air is maintained at 30ºC and the temperature of the body cools from 80ºC to 60ºC in 12 minutes, find the temperature of the body after 24 minutes. 7

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