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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, B.E. - Make-up Examination; July - 2016

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find the nth derivatives of, (i) $\cos x \cos 3x$ (ii) $e^{2x}cos^2x$. 6

b. Find the nth derivative of, $\dfrac{1}{1-x-x^2+x^3}$. 7

c. If $y=\sin^{-1}x$, Prove that $\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-n^2y_n=0$. 7

2 a. State Lagrange's mean value theorem and, verify the same for $f\left(x\right)=e^{-x}$ in [-1, 1]. 6

b. State Rolle's theorem. Verify the same for $f\left(x\right)=e^x\left(\sin x-\cos x\right)$ in $\left[\pi/4, 5\pi/4\right]$. 7

c. Show that $\log\sec x=\dfrac{x^2}{2}+\dfrac{x^4}{12}+\dfrac{x^6}{45}+.........$ 7

UNIT - II

3 a. Evaluate: (i) $\lim\limits_{x\to 0} x\log x$ (ii) $\lim\limits_{x\to 0}\dfrac{x-\tan x}{x^2}$ 6

b. Find the values of 'a' and 'b' if,

$$\lim_{x\to 0}\frac{x\left(1+a\cos x\right)-b\sin x}{x^3}=1$$ 7

c. Show that the polar curves $r=4\sec^2\left(\theta/2\right)$ and $r=9\cos ec^2\left(\frac{\theta}{2}\right)$. 7

4 a. Find the pedal equation of the curve $r\left(1-\cos\theta\right)=2a$. 6

b. Find the radius of curvature for the curve $x^2y=a\left(x^2+y^2\right)$ at the point $\left(-2a, 2a\right)$. 7

c. Show that the radius of curvature for the curve $x=a\log\left(\sec t+\tan t\right)$, $y=a\sec t$ *is* $a\sec^2 t$. 7

UNIT - III

5 a. If $u=\tan^{-1}\left(y/x\right)$ verify that $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{\partial^2 u}{\partial y\partial x}$ 6

b State Euler's theorem. Using the same, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$,

where $u=\sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ 7

Contd…..2

c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ 7

6 a. A particle moves along the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is time. Find the components of velocity and acceleration at t = 1, in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$. 6

b. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along $2I - 3J + 6K$. 7

c. Show that $\vec{F} = (y+z)I + (z+x)J + (x+y)K$ is irrotational. Also, find a scalar function ϕ such that $\vec{F} = \nabla\phi$. 7

UNIT - IV

7 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x dx$, where n is a positive integer. 6

b. Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x dx$ using reduction formula. 7

c. Trace the curve: $r = a(1+\cos\theta)(a > 0)$. 7

8 a. Find the area of an arch of the cycloid: $x = a(\theta - \sin\theta),\ \ y = a(1-\cos\theta),\ \ 0 \le \theta \le 2\pi.$ 6

b. Find the perimeter of the cardiod : $r = a(1+\cos\theta)$. 7

c. Assuming the validity of the differentiation under integral sign, evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx (\alpha \ge 0)$ hence evaluate $\int_0^1 \frac{x^4 - 1}{\log x} dx$. 7

UNIT - V

9 a. Solve: $(xy + y^2)dx + (x + 2y - 1)dy = 0$. 6

b. Solve: $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - x/y\right)dy = 0$. 7

c. Solve: $(x - 4y - 9)dx + (4x + y - 2)dy = 0$. 7

10a. Solve: $\cos y dy + \left[(\sin y - \sin x)\cos x\right]dx = 0$. 6

b. Find the orthogonal trajectories of the family of curves $r = 4a\sec\theta\tan\theta$. 7

c. The R-L series circuit differential equation acted by an electromotive force $E\ sin\omega t$ satisfies $L\frac{di}{dt} + Ri = E\sin\omega t$, If there is no current in the circuit initially, obtain the value of the current at any time t. 7

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