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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, B.E. - Make-up Examination; Jan/Feb - 2017

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find the n^{th} derivative of,

i) $\log_{10}\left\{(1-2x)^3(8x+1)^5\right\}$ 6

ii) $e^{2x}\cos^3 x$.

b. Find the n^{th} derivative of, $\dfrac{x}{1+3x+2x^2}$. 7

c. If $y=\dfrac{\sinh^{-1}x}{\sqrt{1+x^2}}$, Prove that $\left(1+x^2\right)y_{n+2}+(2n+3)xy_{n+1}+(n+1)^2y_n=0$. 7

2 a. State Cauchy's mean value theorem and verify the same for the functions, $f(x)=\sin x$ and $g(x)=\cos x$ in $[a,b]$. 6

b. State Rolle's theorem verify the same for $f(x)=e^x(\sin x-\cos x)$ in $\left[\dfrac{\pi}{4},\dfrac{5\pi}{4}\right]$. 7

c. Using Maclaurin's series, expand $f(x)=\log(\sec x)$ upto the term containing x^4. 7

UNIT - II

3 a. Evaluate: i) $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x}$ ii) $\lim\limits_{x\to 0}\dfrac{x^2+2\cos x-2}{x\sin^3 x}$. 6

b. Find the value of the constant 'a' such that $\lim\limits_{x\to 0}\dfrac{\sin 2x+a\sin x}{x^3}$ is finite. What is the finite limit? 7

c. Show that the pairs of cures $r^n=a^n\cos n\theta$ and $r^n=b^n\sin n\theta$ intersect each other orthogonally. 7

4 a. Find the Pedal equation of the curve $\dfrac{2a}{r}=(1+\cos\theta)$. 6

b. Find the radius of curvature for the curve $x^3+y^3=3axy$ at the point $\left(\dfrac{3a}{2},\dfrac{3a}{2}\right)$ on it. 7

c. Find the radius of curvature for the curve $x=a\log(\sec t+\tan t)$, $y=a\sec t$. 7

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UNIT - III

5 a. If $u = \tan^{-1}\left[\frac{xy}{\sqrt{1+x^2+y^2}}\right]$, Show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{\left(1+x^2+y^2\right)^{3/2}}$. 6

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, Prove that

i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$. 7

c. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$ show that

$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$. 7

6 a. A particle moves along the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$ where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$. 6

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2\hat{i} - \hat{j} - 2\hat{k}$. 7

c. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ where $\vec{F} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right)$. 7

UNIT - IV

7 a. Obtain the reduction formula for $\int \sin^n x\,dx$ and $\int_0^{\pi/2} \sin^n x\,dx$ where n is a positive integer. 6

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$ using reduction formula. 7

c. Trace the curve: $y^2(2a - x) = x^3,\ \ a > 0$. 7

8 a. Find the length of an arch of the cycloid $x = a(\theta - \sin\theta),\ \ y = a(1 - \cos\theta)$. 6

b. Find the surface area of the revolution of the curve $r = a(1 + \cos\theta)$ about the initial line. 7

c. Evaluate: $\int_0^{\infty} e^{-\alpha x}\frac{\sin x}{x}dx$ by differentiating under the integral sign. 7

UNIT - V

9 a. Solve: $x^2y\,dx - \left(x^3 + y^3\right)dy = 0$. 6

b. Solve: $y(2x - y + 1)\,dx + x(3x - 4y + 3)\,dy = 0$. 7

c. Solve: $\frac{dy}{dx} + y\cot x = 4x\cos ecx,$ if $y = 0$ when $x = \frac{\pi}{2}$. 7

10 a. Solve: $x^3\frac{dy}{dx} - x^2y = -y^4\cos x$. 6

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self orthogonal. 7

c. If the temperature of the air is 30ºC and a metal ball cools from 100ºC to 70ºC in 15 minutes. Find how long will it take for the metal ball to reach a temperature of 40ºC. 7

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