



**P.E.S. College of Engineering, Mandya - 571 401**

*(An Autonomous Institution affiliated to VTU, Belgaum)*

**Third Semester, B.E. - Make-up Examination; Jan/Feb - 2017**

**Engineering Mathematics - III**

**(Common to all Branches)**

*Time: 3 hrs*

*Max. Marks: 100*

**Note: Answer FIVE full questions, selecting ONE full question from each unit.**

**UNIT - I**

1 a. Find the missing values in the following table :

|        |   |   |    |   |     |
|--------|---|---|----|---|-----|
| $x$    | 1 | 2 | 3  | 4 | 5   |
| $u(x)$ | 8 | ? | 64 | ? | 216 |

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b. Find the number of students who have obtained

i) Less than 45 marks    ii) Between 40 and 45 marks using the following data.

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|                 |         |         |         |         |         |
|-----------------|---------|---------|---------|---------|---------|
| Marks:          | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | 70 - 80 |
| No. of Students | 31      | 42      | 51      | 35      | 31      |

c. Using Newton's general interpolation formula, fit an interpolating polynomial for the data :

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|        |    |   |   |    |
|--------|----|---|---|----|
| $x$    | -3 | 0 | 1 | 3  |
| $f(x)$ | 2  | 1 | 0 | -1 |

2 a. Use Lagrange's interpolation formula, find  $f(10)$ , using the following data :

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|       |    |    |    |    |
|-------|----|----|----|----|
| $x$ : | 5  | 6  | 9  | 11 |
| $y$ : | 12 | 13 | 14 | 16 |

b. Use Stirling's interpolation formula to find  $Y_{35}$ , given  $Y_{20} = 512$ ,  $Y_{30} = 439$ ,  $Y_{40} = 346$  and  $Y_{50} = 243$ .

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c. Find  $f(2.5)$  by using Bessel's interpolation formula given that :

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|        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| $x$    | 0      | 1      | 2      | 3      | 4      |
| $f(x)$ | 7.4720 | 7.5854 | 7.6922 | 7.8119 | 7.9252 |

**UNIT - II**

3 a. Given the data :

|        |   |    |    |     |     |
|--------|---|----|----|-----|-----|
| $x$    | 0 | 2  | 4  | 6   | 8   |
| $f(x)$ | 7 | 13 | 43 | 145 | 367 |

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Compute  $f'(7)$ .

b. Find  $f'(6)$  and  $f''(6)$  from the following data :

|        |   |    |    |     |     |     |
|--------|---|----|----|-----|-----|-----|
| $x$    | 0 | 2  | 3  | 4   | 7   | 9   |
| $f(x)$ | 4 | 26 | 58 | 112 | 466 | 922 |

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Using Newton's divided difference formula.

c. Find the maximum and minimum value of  $y$  from the following data :

|     |   |       |   |       |    |
|-----|---|-------|---|-------|----|
| $x$ | 0 | 1     | 2 | 3     | 4  |
| $y$ | 0 | -0.25 | 2 | 15.75 | 56 |

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4 a. Use Boole's formula to evaluate  $\int_1^5 e^{1/x} dx$ , taking  $h = 1$ .

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b. Use Simpson's  $\left(\frac{3}{8}\right)^{th}$  rule, evaluate  $\int_0^{0.3} \sqrt{1-8x^3} dx$  by considering three equal intervals.

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c. Using Weddle's rule, evaluate taking 6 equal parts,  $\int_4^{5.2} \log_e x dx$ .

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**UNIT - III**

5 a. Find the Fourier series of  $f(x)$ , if  $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$

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b. Expand  $f(x) = x - x^2$  in terms of Fourier series valid in  $-\pi < x < \pi$ .

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c. Find the half-range cosine series of  $f(x) = (x-1)^2$  valid in  $0 \leq x \leq 1$ . Hence, deduce that,

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$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

6 a. Obtain the complex form of the Fourier series for  $f(x) = x$  in  $-\pi \leq x \leq \pi$ .

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b. Obtain the Fourier series of  $f(x) = |x|$  in  $[-l, l]$ .

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c. Express  $y$  as a Fourier series upto second harmonics, given :

|     |      |         |          |       |          |          |        |
|-----|------|---------|----------|-------|----------|----------|--------|
| $x$ | 0    | $\pi/3$ | $2\pi/3$ | $\pi$ | $4\pi/3$ | $5\pi/3$ | $2\pi$ |
| $y$ | 1.98 | 1.30    | 1.05     | 1.30  | -0.88    | -0.25    | 1.98   |

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**UNIT - IV**

7 a. Find the Fourier transform of,  $f(x) = \begin{cases} 1, & \text{for } |x| \leq 1 \\ 0, & \text{for } |x| > 1 \end{cases}$  and hence evaluate  $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$ .

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b. Find the Fourier sine transform of  $e^{-|x|}$ . Hence show that:  $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ ,  $m > 0$ .

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c. Show that  $f(x) = xe^{-x^2/2}$  is self-reciprocal under Fourier cosine transform.

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8 a. Find the Z-transforms of  $\cos n\theta$  and  $\sin n\theta$ .

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b. Obtain the inverse Z-transforms of  $\frac{z^3 - 20z}{(z-2)^3(z-4)}$ .

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c. Solve,  $y_{n+2} - 4y_n = 0$  with  $y_0 = 0$ , and  $y_1 = 2$ , using Z-transforms.

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## UNIT - V

- 9 a. Form the partial differential equation by eliminating the arbitrary function in  $f(x^2 + y^2, z - xy) = 0$ . 6
- b. Solve  $(\partial u / \partial x) = 2(\partial u / \partial t) + u$  with  $u(x, 0) = 6e^{-3x}$ , by using the method of separation of variables. 7
- c. Solve:  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ . 7
- 10 a. Obtain the various possible solutions of  $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ , by the method of separation of variable. 10
- b. Solve the heat equation  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  with boundary conditions, 10  
 $u(x, 0) = 3 \sin n\pi x$  and  $u(0, t) = 0 = u(1, t)$ .

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