***U.S.N*** | | | | | | | | | | |

# P.E.S. College of Engineering, Mandya - 571 401

***(An Autonomous Institution affiliated to VTU, Belgaum)***

**Fourth Semester, B.E. – Make-up Examination, July/Aug. - 2015**

## Engineering Mathematics - IV

**(Common to AU, CV, ME & IP Branches)**

*Time: 3 hrs* *Max. Marks: 100*

***Note**: Answer any **FIVE** full questions selecting **ONE** full question from **each unit**.*

**UNIT - I**

1\. a. Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$ — 6

b. Find the analytic function whose real part is

$$\frac{\sin 2x}{\cosh 2y - \cos 2x}$$ — 7

c. Find the Bilinear transformation which map the points *z = 1, i, -1 into* w = *i, 0, - i.* — 7

2 a. Evaluate $\int_0^{2+i} (\bar{z})^2\, dz$ — 6

(i) along the line *x = 2y* (ii) the real axis upto 2 and then vertically to $2+i$

b. Expand $f(z) = \dfrac{z+1}{(z+2)(z+3)}$ in a Laurentz's series. — 7

Valid for (i) $|z| > 3$ ii) $2 < |z| < 3$

c. Evaluate $\int_C \dfrac{(z^2+5)}{(z-2)(z-3)} dz$. using residue theorem where $C : |z| = 4$ — 7

**UNIT - II**

3 a. Using Newton-Raphson iterative formula find the real root of the equation $x \log_{10} x = 1.2$ correct to four decimal places. — 6

b. Find the smallest root of the equation $x^3 - 6x^2 + 11x - 6 = 0$ using Ramanujan's method. — 7

c. Find the real root of the equation $\cos x + 1 = 3x$ correct to three decimal places using regula-falsi method. — 7

4 a. Use Taylor's series method to find *y* at *x = 0.1*. Consider the terms upto third degree term given that $\dfrac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ — 6

b. Use Runge-Kutta method of fourth order to solve $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$ with *y*(0) = 1 at *x* = 0.2. — 7

Contd…2

c. Using Milne's predictor – corrector method find the value of $y(0.8)$, Given that $\frac{dy}{dx} = x - y^2, y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,\ y(0.6) = 0.1762$ 7

**UNIT - III**

5 a. The first four moments about an arbitrary value '5' of a frequency distribution $\{x_i, f_i\}$, $i$ = 1, 2, 3,… are 2, 20, 40 and 50. Find the Skewness and Kourtosis. 6

b. Fit the parabola of the form $y = a+bx+cx^2$ for the data 7

| | | | | | |
|---|---|---|---|---|---|
| *x* | 0 | 1 | 2 | 3 | 4 |
| *y* | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

c. Obtain the lines of regression and hence find the correlation coefficient of the data. 7

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *x* | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| *y* | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

6 a. A random Variable *x (= x)* has the following Probability distribution for various values of *x*. 6

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *x* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| *P(x)* | k | 3k | 5k | 7k | 9k | 11k | 13k |

i) find k ii) Evaluate $P(x \geq 5)$ and $P(3 < x \leq 6)$

b. The number of Telephone lines busy at a particular time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are selected at random what is the probability that 7
i) no line is busy ii) at least one line is busy iii) at most 2 lines are busy.

c. The marks of 1000 students in an examination fallows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be 7
i) Less than 65 ii) more than 75 iii) between 65 and 75 (Given $\phi$ (1) = 0.3413)

**UNIT - IV**

7 a. Define : i) Standard error ii) Type – I and Type – II errors. 6

b. A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800. It was revealed that 180 families were illiterates. Find the probable limits of illiterate families in a population of 2000. 7

c. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches (value of $t_{0.05}$ = 2.262 for 9 d.f) 7

8 a. Solve the following system by Gauss-Seidal method;
$8x+3y+2z=13;\ 2x+y+6z=9;\ x+5y+z=7;$ starting from $[0, 0, 0]^T$ (carry out 3 iterations.) 6

Contd…3

b. Solve by relaxation method, 7

$$8x - y + z = 18$$
$$2x + 5y - 2z = 3$$
$$x + y - 3z = -6$$

c. Using the power method, find the largest eigen value and corresponding eigen vector of the matrix, 7

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Taking $[1\ 1\ 1]^T$ as the initial vector. (Perform 5 iterations.)

**UNIT - V**

9 a. Find the function $y$ which makes the integral $\int_{x_1}^{x_2} \left(1 + xy' + xy'^2\right) dx$ an extremum. 6

b. Solve the variational problem: 7

$$\delta \int_0^{\pi/2} \left(y^2 - y'^2\right) dx = 0; \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 2$$

c. Find the curve passing though the points $(x, y)$ & $(x_2, y_2)$ to have minimum surface area of revolution when it is rotated about $x$-axis. 7

10 a. Define; i) Mean failure rate ii) Mean time to failure iii) Mean time between failure 6

b. Define the exponential failure law, obtain the linear failure model for the following data by the least square sense and hence find the mean time failure. 7

| | | | | | | |
|---|---|---|---|---|---|---|
| T | 5 | 10 | 15 | 20 | 25 | 30 |
| Z | 1.2 | 2.2 | 3.2 | 4.2 | 5.2 | 6.2 |

c. Define the Weibull failure law. The failure rate with respect to time as per the test results fallows to following Weibull failure law 7

i) $z(t) = \dfrac{1}{\sqrt[3]{t}}$

ii) $z(t) = \dfrac{3}{t^{3/5}}$. Compute the MTTF in each case.

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