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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Make-up Examination; Jan/Feb - 2017

Engineering Mathematics - IV

(Common to AU, CV, ME & IP Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: i) Answer **FIVE** full questions, selecting **ONE** full question from each unit.*
ii) Use of statistical tables is allowed.

UNIT - I

1 a. Show that: $f(z) = z^n$, where n is positive integer, is analytic and hence find its derivative. 6

b. Find the analytic function $f(z)$ as a function of z given that $u + v = x^3 - y^3 + 3xy(x - y)$. 7

c. Find the bilinear transformation that transforms the points i, 1, -1 onto the points 1, 0, ∞ respectively. 7

2 a. Expand: $f(x) = \dfrac{z-1}{z+1}$ in Taylor's series about the point z = 1. 6

b. Using Cauchy integral formula, evaluate the integral : $\oint_c \dfrac{e^{\pi z}}{(z^2+1)^2}$, where c is the circle $|z - i| = 1$. 7

c. Using Cauchy's Residue theorem, evaluate $\int_c \dfrac{dz}{z^3(z-1)}$ where c is the circle : $|z| = 2$. 7

UNIT - II

3 a. Find a root of the equation $xe^x = \cos x$ using Regula-Falsi method. Carryout two iterations. 6

b. Using Newton-Raphson method, find the root that lies near x = 4.5, of the equation tan$x$ = x. Correct to four decimal places. 7

c. Find a real root of the equation $x^3 - x - 1 = 0$, using fixed point iterations method. Accelerate the convergence by Aitken's Δ^2 – method (Carryout three iterations). 7

4 a. Use Taylor's series method to find y and x = 0.1 given that $\dfrac{dy}{dx} = x^2 + y^2, y(0) = 1$ (Considering the terms upto fourth derivative). 6

b. Using fourth-order Runge-Kutta method, solve $y' = x + y^2$, $y(0) = 1$ at $x = 0.2$ in steps 0.1. 7

c. Use Milne's predictor-corrector method to find y at x = 0.8 given

$$\frac{dy}{dx} = x - y^2 \text{ with } y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.07,\ y(0.6) = 0.17$$ 7

(Use corrector formula twice).

Contd......2

UNIT - III

5 a. The first four moments about an arbitrary value '5' of a frequency distribution $\{x_i, f_i\}$ $i = 1$, 2, 3, ….. are 1, 4, 10 and 46. Compute the first four moments about the mean. Also compute skewness and kurtosis. 6

b. Fit a curve of the form $y = ab^x$ for the data given below : 7

x:	2	4	6	8	10	12
y:	1.8	1.5	1.4	1.1	1.1	0.9

c. Obtain the lines of regression and hence find the coefficient of correlation for the data. 7

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

6 a. A random variable X (=x) has the following probability function for various value of x. 6

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

i) Find K ii) Evaluate P ($x < 6$) and P ($3 < x \leq 6$).

b. The probability that a rivet manufactured by a factory be defective is $\frac{1}{10}$. If 12 rivets are manufactured, what is probability that, 7

i) Exactly two will be defective ii) At least two will be defective

iii) None will be defective.

c. The number of accidents in a year by a taxi driver in a city follows Poisson's distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with, 7

i) No accidents in a year ii) More than three accidents in a year.

UNIT - IV

7 a. Results extracts revealed that in a certain school over a period of five years 725 students had passed and 615 students had failed. Test the hypothesis that success and failure are in equal proportions. 6

b. A random sample of 10 boys had the following I.Qs : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. 7

c. A dice is thrown 264 times and the number appearing on the faces (x) follows the following frequency distribution. 7

x	1	2	3	4	5	6
f	40	32	28	58	24	60

Calculate the value of ψ^2.

Contd.....3

8 a. Employing Gauss-Siedal method, solve the system of equation : 6

$10x+y+z=12,\ 2x+10y+z=13,\ 2x+2y+10z=14.$ Carryout 3 iterations.

b. Solve the following equation using relaxation method, 7

$10x-2y-2z=6,\ -x+10y-2z=7,\ -x-y+10z=8.$

c. Find the dominant Eigen value and the corresponding Eigen vector of the matrix.

$$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ 7

by power method taking $X^{(0)}=[1,1,1]^T$. Perform 5 iterations.

UNIT - V

9 a. With usual notations establish the Euler's equation as $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0.$ 6

b. Find the extremal of the functional, $\int_0^{\frac{\pi}{2}}\left[\left(y'\right)^2-y^2+4y\cos x\right]dx,\ y(0)=y\left(\frac{\pi}{2}\right)=0.$ 7

c. Define a geodesic. Find the geodesic on a surface given that the arc length on the surface is,

$$s=\int_{x_1}^{x_2}\sqrt{x\left(1+y'^2\right)}\,dx.$$ 7

10 a. Define : 6

i) Failure density ii) Failure rate iii) Mean failure rate.

b. Define the Weibull failure law. The life time of a system ϕ modeled as a Weibull distribution with reliability $R(t)=e^{-\alpha t^2}$, where α is a positive constant. It is observed that 15% of the systems that have lasted 90 hours fail before 100 hours. Determine the value of α. 7

c. Compute the mean time to failure density and mean failure rate for the following data : 7

Time interval in hours	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
No. of Failure	3	16	26	31	16	8

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