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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. – Semester End Examination; June - 2016

Engineering Mathematics - IV

(Common to AU, CV, ME & IP Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: i) Answer **FIVE** full questions, selecting **ONE** full question from each Unit.*
ii) Use of statistical tables is allowed.

UNIT - I

1. a. Show that $f(z) = \sin Z$ is analytic. Hence find its derivative. 6

b. Find an analytic function $f(z) = u + iv$, given that $u + v = \frac{1}{r^2}(\cos 2\theta - \sin 2\theta)$, $r \neq 0$. 7

c. Discuss the transformation $W = Z^2$. 7

2 a. Evaluate: $\int_{1-i}^{2+i} (2x + iy + 1)\, dz$ along $x = t + 1,\ y = 2t^2 - 1$. 6

b. Obtain Laurent's expansion of $f(z) = \frac{z+3}{z(z^2 - z - 2)}$ in the regions, 7

(i) $|z| < 1$ (ii) $1 < |z| < 2$

c. Evaluate :

$\int_C \frac{dz}{z^3(z-1)}$ Where C is the circle $|z| = 2$ using Cauchy's residue theorem. 7

UNIT - II

3 a. Find by Regula-Falsi method the real root of the equation $\log_e x - \cos x = 0$ that lies in (1, 1.5) correct to four places of decimals. Carry out three iterations. 6

b. Use Newton-Raphson method to find a real root of $x^4 - 3x^3 + 2x^2 + 2x - 7 = 0$ that lies near 2.1 upto four decimal places. 7

c. Find a real root of the equation $x^3 - x - 1 = 0$ using fixed point iteration method. Accelerate the convergence by Aitken's Δ^2 – method (carry out three iterations). 7

4 a. Find by Taylor's series method the value of the y at $x = 0.1$ and $x = 0.2$ to five places of decimals from $\frac{dy}{dx} = x^2 y - 1;\quad y(0) = 1$ (Considering terms upto third degree). 6

b. Using Runge-Kutta method of fourth order, solve

$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2},\ y(0) = 1\ \ at\ x = 0.2\ and\ 0.4$ 7

c. Find $y(2)$, if $y(x)$ is the solution of $\frac{dy}{dx} = \frac{x+y}{2}$ given that $y(0) = 2$, $y(0.5) = 2.636$, $y(1.0) = 3.595$ *and* $y(1.5) = 4.968$ using Milne's predictor and corrector method to four decimal places. 7

UNIT - III

5 a. Fit a least square geometric curve $y = ax^b$ to the following data, 6

x:	1	2	3	4	5
y:	0.5	2	4.5	8	12.5

b. Given that the four moments about 2 are 1, 25, -30 and 75. Find the skewness and kurtosis based on moments. 7

c. For the regression lines $x = y$ *and* $4x - y = 3$, find; 7

(i) Mean values of *x* and *y*

(ii) Coefficient of correlation

(iii) Variance in *y*, given that the variance in *x* is 1.

6 a. From the sealed box containing a dozen apples it was found that 3 apples are perished. Obtain the probability distribution of the number of perished apples when 2 apples are drawn at random. Also find the mean and standard deviation of this distribution. 6

b. In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing, 7

(i) No defective (ii) One defective

(iii) Two defective blades in a consignment of 10,000 pockets.

c. The life of a certain type of electric lamps is normally distributed with mean 2040 hours and standard deviation 60 hours. In a consignment of 3000 lamps, how many would be expected to burn for, 7

i) More than 2150 hours ii) Less than 1950 hours.

Given $P(0 \le z \le 1.83) = 0.4664$; $P(0 \le z \le 1.33) = 0.4082$.

UNIT - IV

7 a. Define : 6

i) Testing of hypothesis ii) Standard error iii) Type -I and Type-II errors.

b. A random sample of 10 boys had the following I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie. 7

Contd 3

c. Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05. 7

x:	0	1	2	3	4
f:	419	352	154	56	19

8 a. Solve by Gauss-Seidel iteration method, the equations $20x + y - 2z = 17;\quad 3x + 20y - z = -18;\quad 2x - 3y + 20z = 25$, Carry out three iterations. 6

b. Solve by Relaxation method, 7

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$, by power method, taking $[1, 0, 0]^T$ as the initial Eigen vector (Perform five iterations). 7

UNIT - V

9 a. Derive Euler's equation in the usual form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0$. 6

b. Find the Extremel of the functional $\int_0^{\pi/2} \left(y'^2 - y^2 + 4y\cos x\right) dx;\quad y(0) = y(\pi/2) = 0$. 7

c. Find the path on which a particle in the absence of friction will slide from one point to another point in the shortest time under the action of gravity. 7

10a. Define : 6

i) Reliability ii) Mean time to failure iii) Failure rate.

b. Define the Weibull failure law. The life time of a system is modeled as a Weibull distribution with reliability $R(t) = \exp(-\alpha t^2)$, where α is a positive constant. It is observed that 15% of the systems that have lasted 90 hours fail before 100 hours. Determine the value ofα. 7

c. Suppose that the life time of a device is exponentially distributed. If the reliability of the device for a 100 hour period of operation is 0.90, how many hours of operation may be considered to achieve a reliability of 0.95? 7

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