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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. – Semester End Examination; June / July -2015

Engineering Mathematics – IV

(Common to AU, CV, ME & IP Branches)

Time: 3 hrs *Max. Marks: 100*

***Note** : i) Answer **FIVE** full questions, selecting **ONE** full question from each Unit.*
ii) Use of statistical tables is allowed.

UNIT - I

1. a. Show that $f(z) = z + e^z$ is an analytic function. Hence find its derivative. 6

b. Find an analytic function $f(z) = u + iv$, given that $u = x\sin x\cosh y - y\cos x\sinh y$. 7

c. Find the bilinear transformation which maps the points $z = 0, -i, 2i$ into $\omega = 5i, \infty, -i/3$ respectively. What are the invariant points in this transformation? 7

2 a. Evaluate: $\int_C z^2 dz$ along the curve C made up of two line segments, one from $z = 0$ to $z = 3$ and another from $z = 3$ to $z = 3+i$. 6

b. Expand $f(z) = \dfrac{z}{(z-1)(z-3)}$ as a Laurent's series valid for i) $1 < |z| < 3$ ii) $|z-1| < 2$ 7

c. Use Cauchy's residue theorem to evaluate,

$\int_c \dfrac{e^z dz}{z^2 + 4}$ where C is the circle $|z - i| = 2$ 7

UNIT - II

3 a. Apply regula-falsi method to find a real root of the equation $\tan x + \tanh x = 0$ that lies between 2 and 3. Carryout three iterations. 6

b. Find the smallest root of the equation, $x^3 - 9x^2 + 26x - 24 = 0$, using Ramanujan's method. 7

c. Apply Newton Raphson method to find a real root of the equation $x\log_{10} x = 1.2$ correct to four decimal places. 7

4 a. Use Runge – Kutta method of order IV to find $y(1.1)$, given that $\dfrac{dy}{dx} = xy^{1/3}, y(1) = 1$ *taking* $h = 0.1$ 6

b. Apply modified Euler's method to find y at $x = 0.2$, given $\dfrac{dy}{dx} = 3x + y/2$, $y(0) = 1$ *taking* $h = 0.1$ Perform three iterations at each stage. 7

c. Given $\dfrac{dy}{dx} = x - y^2$ and the data $y(0) = 0,\ y(0.2) = 0.02,\ y(0.4) = 0.0795,\ y(0.6) = 0.1762$ compute y at $x = 0.8$ by using Adam-Bashforth predictor-corrector method. 7

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UNIT – III

5 a. The first four moments about an arbitrary value "4" of a frequency distribution are -1.5, 17, -30 and 108. Find the skewness and kurtoses based on moments. 6

b. Fit a curve of the form $y = ab^x$ for the data 7

X:	1	2	3	4	5	6	7
Y:	87	97	113	129	202	195	193

And hence find the estimation of y when $x = 8$.

c. If θ is the angle between the lines of regression, show that $\tan\theta = \frac{\sigma x \sigma y}{\sigma x^2 + \sigma y^2}\left(\frac{1-r^2}{r}\right)$.Explain the significance when r = ±1. 7

6 a. A random variable X $(= x)$ has the following probability function for various values of x: 6

X:	0	1	2	3	4	5	6	7
Y:	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

Find

i) the value of K

ii) $P(x < 6)$

iii) $P(3 < x \le 6)$

b. The number of accidents in a year to taxi drivers in a city follows a Poisson's distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with (i) no accidents (ii) More than 3 accidents in a year. 7

c. The life of an electric bulb is a normal variable with mean life of 2040 hours and standard deviation of 60 hours. Find the probability that a randomly selected bulb will burn for 7

i) more than 2150 hours ii) less than 1950 hours.

Given $\phi(1.83) = 0.4664$ and $\phi(1.5) = 0.4332$

UNIT – IV

7 a. Define 6

i) Standard error

ii) Type – I and Type – II errors.

b. A die is tossed 960 times and it falls with 5 upwards 184 times. Is the die biased at 5% level of significance? 7

c. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71 and 71. Test the hypothesis that mean height of the universe is 66 inches. (Use $t_{0.05}(9) = 2.262$). 7

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8 a. Solve the system of equations $6x+15y+2z=72,\ 27x+6y-z=85,\ x+y+54z=110$ by Gauss – Saidal method to obtain a numerical solution correct to three decimal places of accuracy. [Take ($x^{(0)}$, $y^{(0)}$, $z^{(0)}$ = (0, 0, 0)] 6

b. Solve by Relaxation method 7

$$\begin{aligned} 10x-2y-2z&=6\\ -x+10y-2z&=7\\ -x-y+10z&=8 \end{aligned}$$

c. Use Rayleigh's power method, find numerically the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{pmatrix} 25 & 1 & 2\\ 1 & 3 & 0\\ 2 & 0 & -4 \end{pmatrix}$, taking initial Eigen vector as $[1, 0, 0]^T$ 7

UNIT – V

9 a. With usual notation establish the Euler's equation as, $\dfrac{\partial f}{\partial y}-\dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y^1}\right)=0$ 6

b. Solve the variational problem: $\delta\int_0^{\pi/2}\left(y^2-(y')^2\right)dx;\ \ y(0)=0, y\left(\pi/2\right)=2$. 7

c. Define a geodesic. Find the geodesic on the surface of a plane. 7

10 a. Explain; i) system reliability ii) mean time to failure. 6

b. Define the normal failure law. Suppose that the life time of a system is normally distributed with standard deviation equal to 10 hours. If the system has a reliability of 0.99 for an operation period of 100 hours, what should its expected life be? 7

c. Define the Weibull failure law. Find the mean time to failure for a Weibull distribution. 7

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