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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E.: Make – up Examination; Jan/Feb. - 2016

Engineering Mathematics - IV

(Common to E&CE, E&EE, CS&E and IS&E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. Use Newton - Raphson method to find the root of $\sqrt[3]{37}$ correct to three decimal places (carry out 3 iterations). 6

b. Use the Regula - Falsi method to find a real root of the equation $\cos x = 3x - 1$ correct three decimals. (Carryout three iterations). 7

c. Find a real root of the equation $x^3 - x - 1 = 0$ in the interval (1, 2) using fixed point iteration method and accelerate it by Aitken's Δ^2 – process. 7

2 a. Using modified Euler's method to find y at x = 0.2 given $\frac{dy}{dt} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking h = 0.1 perform three iterations at each step. 6

b. Using Runge - Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.1$ 7

c. Apply Adams-Bashforth method to solve the equation $(y^2+1)dy - x^2dx = 0$ at x = 1 given $y(0)=1$, $y(0.25) = 1.0026$, $y(0.5) = 1.0206$ and $y(0.75) = 1.0679$ 7

UNIT - II

3 a. Show that $f(z) = z^n$, where n is positive integer, is analytic and hence find its derivative. 6

b. Find the analytic function whose real part is $u = \frac{x^4 - y^4 - 2x}{x^2 + y^2}$. Hence determine v. 7

c. Find the bilinear transformation which maps z = ∞, i, 0 into w = -1, - i, 1. Also find its invariant points. 7

4 a. State Cauchy's integral formula and use it to evaluate $\int_c \frac{dz}{z^2 - 4}$ where c represents the circle $|z| = 3$ 6

b. Expand $f(z) = \frac{z-1}{(z-2)(z-3)^2}$ as s Laurent's series valid for, 7

i) $|z| > 3$ ii) $2 < |z| < 3$

Contd…..2

c. Using Cauchy's residue theorem evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where c: |z| = 3. 7

UNIT - III

5 a. The first four moments about an arbitrary value '5' of a frequency distribution $\{x_i f_i\}_i = 1, 2, ...n$ are 2, 20, 40, and 50. Find skewness and Kurtosis based on moments. 6

b. Fit a second degree parabola $y = a + bx + cx^2$ 7

X:	-2	-1	0	1	2
Y:	-3.150	-1.390	0.620	2.880	5.378

c. Find the correlation coefficient and the equation of the lines of regression for the following values of *x* and *y*. 7

X:	1	2	3	4	5
Y:	2	5	3	8	7

6 a. The p.d.f. of a variate *x* is given by the following table. 6

x:	0	1	2	3	4	5	6
P(*x*)	K	3K	5K	7K	9K	11K	13K

For what value of K. Also find $P(x \geq 5)$ and $P(3 < x \leq 6)$

b. The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1 that a line is busy. If 10 lines are chosen at random. What is the probability that,

(i) no line is busy

(ii) all lines are busy 7

(iii) atleast one line is busy

(iv) atmost 2 lines are busy.

c. In a test an electric bulbs, it has found that the life time of a particular brand was distributed normally with an average life of 2000 hours and S.D. of 60 hours. In a purchase of 2500 bulbs, find the number of bulbs that are likely to last for (i) more than 2100 hours. (ii) less than 1950 hours (iii) between 1900 to 2100 hours. 7

Given $\phi(1.67) = 0.4525$ $\phi(0.83) = 0.2967$.

UNIT - IV

7 a. The joint distribution of two random variables X and Y is as follows. 6

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute; (i) E(X) and E(Y) (ii) E(XY) (iii) COV (X, Y)

Contd…..3

b. Find the unique fixed probability vector for the regular stochastic matrix. 7

$$\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

c. The joint density function of two continuous random variables X and Y is given by, 7

$$f(x) = \begin{cases} kxy & 0 \le x \le 4 \quad 1 < y < 5 \\ 0 & \text{Otherwise} \end{cases}$$

Find; i) the value of K ii) $E(x\,y)$ iii) $E(2x+3y)$

8 a. Given the date of stochastic process defined on a finite sample space with three sample points $X(t,\lambda_1)=3,\ X(t,\lambda_2)=3\cos t$ and $X(t,\lambda_3)=3\sin t$, where; $P(\lambda_1)=P(\lambda_2)=P(\lambda_3)=\frac{1}{3}$ represents the probability of the assignments compute μ(t) and $R(t_1, t_2)$. 6

b. Three boys A, B, C are throwing ball to each other. A always throws the balls to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probability that after three throws i) A has the ball ii) B has the ball iii) C has the ball. 7

c. Define stationary distribution and absorbing state of a Markov chain with examples. 7

UNIT - V

9 a. Define subspace with suitable example. 6

b. Define linear dependent and independent vector space with suitable example. 7

c. Define linear transformation of vector space. Find the rank and nullity of the linear transformation, 7

$T : R^4 \rightarrow R^3$ by

$T(x,y,z,t) = (x-y+z+t, 2x-2y+3z+4t, 3x-3y+4z+5t)$

10a. Solve: $x+y+54z=110$; $27x+6y-z=85$; $6x+15y+2z=72$ using Bans-Seidal iteration method (carry out four iterations). 6

b. Solve: $10x+2y+z=9$; $x+10y-z=-22$; $-2x+3y+10z=22$ using relaxation method. 7

c. Find the dominant Eigen value and the corresponding Eigen vector of the matrix 7

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

By power method. Taking initial Eigen vector [1, 1, 1] perform five iterations.

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