U.S.N | | | | | | | | | | |

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Semester End Examination; June - 2016

Engineering Mathematics - IV

(Common to E&CE, E&EE, CS&E and IS&E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each Unit.*

UNIT - I

1. a. Use Regula-Falsi method to find a real root of the equation $x\log_{10} x - 1.2 = 0$ (Carry out 3 iterations). 6

b. Use Newton-Raphson method to find a real root of the equation $x^2 - 2x - 5 = 0$. Correct to three decimal places. 7

c. Perform two iterations of the linear iteration method followed by one iteration of the Aitken Δ^2 method to find the root of the equation $f(x) = x^3 - 5x + 1 = 0,\ x_0 = 0.5$. 7

2 a. Using modified Euler's method find $y(0.2)$ correct to four decimal places solving the equation, $\frac{dy}{dx} = x - y^2,\ y(0) = 1$ taking h = 0.1. 6

b. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ taking h = 0.2. 7

c. If $\frac{dy}{dx} = 2e^x - y,\ y(0) = 2,\ y(0.1) = 2.010,\ y(0.2) = 2.040,\ y(0.3) = 2.090$ find $y(0.4)$ correct to 4 decimal places by using Adams-Bash forth method. 7

UNIT - II

3 a. Show that $f(z) = \cos hz$ is analytic. Hence find its derivative. 6

b. Construct the analytic function whose imaginary part is $\left(r - \frac{1}{r}\right)\sin\theta,\ r \neq 0$. Hence find the real part of $f(z)$. 7

c. Discuss the transformation $W = Z^2$. 7

4 a. Evaluate $\int_c \bar{z}\, dz$, where c represents the following paths, 6

i) The straight line from $-i$ to i.

ii) The right half of the unit circle $|z| = 1$ from $-i$ to i.

b. Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where c is the circle |z| = 3 by Cauchy's residue theorem. 7

c. Expand $f(z) = \frac{z-1}{(z-2)(z-3)^2}$ as a Laurent's series valid for |z| > 3. 7

UNIT - III

5 a. The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the moments about the mean. Also evaluate β_1 and β_2. 6

b. Fit a second parabola of the form $y = a+bx+cx^2$ to the following data, 7

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

c. Find the correlation coefficient and the equation of the lines of regression for the following values of *x* and *y*, 7

x	1	2	3	4	5
y	2	5	3	8	7

6. a. The probability density function of a variate *x* is given by the following table, 6

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

For what value of k, this represents a valid probability distribution? Also find P($x \geq 5$) and P($3 < x \leq 6$).

b. When a coin is tossed 4 times, find the probability of getting, 7

(i) Exactly one head (ii) At most 3 heads (iii) At least 2 heads.

c. If *x* is an exponential variate with mean 5, evaluate : 7

i) $p(0 < x < 1)$ ii) $p(-\infty < x < 10)$ iii) $p(x \leq 0 \text{ or } x \geq 1)$

UNIT - IV

7 a. A random variable *x* has the following density function, 6

$$p(x) = \begin{cases} kx^2, & -3 \leq x \leq 3 \\ 0, & \text{else where} \end{cases}$$

Evaluate *k* and find; i) $p(1 \leq x \leq 2)$ ii) $p(x \leq 2)$ iii) $p(x > 1)$

b. The joint probability distribution table for two random variable *X* and *Y* as follows: 7

X \ *Y*	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginar probability distribution *X* and *Y*. Also compute (i) Expectations of *X*, *Y* and *XY* (ii) S.D's of *X*, *Y* (iii) Covariance of *X* and *Y*.

c. Find the unique fixed probability vector of the regular stochastic matrix,

$$P = \begin{bmatrix} 0 & ½ & ¼ & ¼ \\ ½ & 0 & ¼ & ¼ \\ ½ & ½ & 0 & 0 \\ ½ & ½ & 0 & 0 \end{bmatrix}$$ 7

8 a. Define :

i) Auto correlation ii) Auto covariance iii) Correlation coefficient. 6

b. Prove that the Markov chain whose transition probability matrix is,

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$ is irreducible? 7

Find the corresponding stationary probability vector.

c. Define the M/M/1 queuing system. In a bus stand there is a single counter for issuing tickets. On an average 12 commuters arrive every 10 minutes. The counter clerk is able to issue 8 tickets in a span of 5 minutes. Find; 7

i) Average number of commuters in the queue ii) Average waiting time in the queue.

UNIT - V

9 a. Define linearly dependent and independent vectors. Prove that,

$u_1 = (1,0,0)$, $u_2 = (0,1,0)$, $u_3 = (0,0,1)$ are linearly independent. 6

b. Determine the set of vectors $u_1 = (1,-1,1)$, $u_2 = (0,1,2)$, $u_3 = (3,0,-1)$ form a basis for R^3. 7

c. Find the matrix of the linear transformation, $T : V_2(R) \to V_3(R)$ such that T(-1, 1) = (-1, 0, 2) and T (2, 1) = (1, 2, 1) 7

10 a. Solve the following system of equations by Gauss-Seidel method :

$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ \ 2x - 3y + 20z = 25.$ 6

b. Solve the following system of equations by Relaxation method :

$12x_1 + x_2 + x_3 = 31,\quad 2x_1 + 8x_2 - x_3 = 24,\quad 3x_1 + 4x_2 + 10x_3 = 58.$ 7

c. Find the largest Eigen value and the corresponding Eigen vector of, $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ 7

By power method taking the initial Eigen vector as $[1, 0, 0]^T$ (Perform 6 iterations).

* * * *