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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Fourth Semester, B.E. - Semester End Examination; June / July -2015

Engineering Mathematics - IV

(Common to EE, EC, CS & E, IS & E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note** : i) Answer **FIVE** full questions, selecting **ONE** full question from each **Unit**.*
ii) Use of statistical tables is allowed.

UNIT – I

1. a. Use Newton – Raphson method to find a real root of the equation $x\log_{10} x = 1.2$ near $x = 2.5$ (Carry out 3 iterations) 6

b. Using Regula – falsi method, Find the fourth root of 32 (carry out four iterations) 7

c. Find a real root of the equation $x^3 - x - 1 = 0$ in the internal (1, 2) using fixed point iteration method and accelerate it by Aitken's Δ^2 – Process. 7

2 a. Use Taylor's series method to obtain a power series in x upto fourth degree terms for the differential equation $\frac{dy}{dx} - 2y = 3e^x$, $y(0) = 0$ Hence find $y(0.1)$ and $y(0.2)$ 6

b. Using modified Euler's method find y at x = 1.2 and x = 1.4 given $\frac{dy}{dx} = 1 + \frac{y}{x}$ with $y(1) = 2$ taking h = 0.2 (perform 3 iterations at each step). 7

c. Apply Milne's method to compute $y(1.4)$ given that $\frac{dy}{dx} = x^2 + \frac{y}{2}$ with $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$ (use corrector formula twice) 7

UNIT – II

3 a. Show that $W = Z^n$ is analytic and hence find its derivative. 6

b. Determine the analytic function $f(z)$. Whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$ 7

c. Discuss the transformation $w = z + \frac{1}{z}$, $z \neq 0$ 7

4 a. State Cauchy's integral formula and use it to evaluation $\int_c \frac{e^{2z}}{(z+1)(z-2)}\, dz$ where c represents the circle |z| = 3. 6

b. Expand $f(z) = \frac{2z+3}{(z+1)(z-2)}$ as a Laurent's series valid for i) $|z| < 1$ ii) $1 < |z| < 2$ 7

c. For the function $f(z) = \frac{z+4}{(z-1)^2(z-2)^3}$. Find the poles and residues. 7

UNIT - III

5 a. The first four moments of a frequency distribution about an arbitrary value are – 1.5, 17, -30 and 108. Find the Skewness and Kurtosis based on moments. 6

b. Fit a parabola of second degree $y=ax^2+bx+c$ in the least square sense for the data. 7

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

c. Find the correlation coefficient and the equation of the lines of regression for the following values of x and y. 7

x	1	2	3	4	5
y	2	5	3	8	7

6. a. A random variable (X = x) has the following probability distribution for various values of x. 6

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find k ii) Evaluate $p(x \geq 6)$ and $p(3 < x \leq 6)$

b. The probability of germination of a seed in a bucket of seeds is found to be 0.7 if 10 seeds are taken for experimenting on germination in a laboratory. 7

Find the probability that (i) exactly eight seeds germinate ii) at least eight seeds germinate.

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and S.D. 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75, Given $\phi(1) = 0.3413$. 7

UNIT - IV

7 a. The joint probability distribution table of two random variable x and y as follows: 6

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find (i) Marginal probability distribution of X and Y (ii) E(X).E(Y) and E(XY)

b. Define the terms (i) Probability vector (ii) Unique fixed probability vector (iii) Stochastic and regular stochastic matrix. Further, Find the unique fixed probability vector of the regular stochastic matrix. 7

$$\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

Contd…3

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws to C. C is just as likely to throw to B as to A. If C was the first person to throw the ball. Find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball. 7

8 a. Given the data of a stochastic process defined on a finite sample space with three sample points $X(t,\lambda_1)=3,\ X(t,\lambda_2)=3\cos t\ \ X(t,\lambda_3)=3\sin t$ where $P(\lambda_1)=P(\lambda_2)=P(\lambda_3)=\frac{1}{3}$ represents the probability of the assignments. Compute $\mu(t)$ and $R(t_1, t_2)$. 6

b. Define; (i) transient state and (ii) absorbing state of a Markov Chain. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night on the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? 7

c. Define the M / M /I queuing system. In a bus stand there is a single counter for issuing tickets. On an average 12 commuters arrive every 10 minutes. The counter clerk is able to issue 8 tickets in a span of 5 minutes. Find i) Average number of commuters in the queue and ii) Average waiting time in the queue. 7

UNIT - V

9 a. Define (i) Vector space and (ii) Subspace with suitable examples. 6

b. i) Define basis of a vector space and ii) Define linearly independent and dependent vectors with suitable examples. 7

c. Define Rank and Nullify of a linear transformation. Find the rack and nullity of the linear transformation T : $V_3(\square) \to V_3(\square)$ by *T (x. y. z) = (x + z, x + y + 2z, 2x + y + 3z)* 7

10 a. Solve: $20x+y-2z=17,\ 3x+20y-z=-18\ \ 2x-3y+20z=25.$ using Gauss – Seidel method (carry out 3 iterations) 6

b. Solve: $5x+2y+z=12,\ \ x+4y+2z=15,\ \ x+2y+5z=20$ using Relaxation method. 7

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix,

$$A=\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$$ 7

Using power method taking the initial Eigen vector as $[1, 0, 0]^T$ (perform six iterations).

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