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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, M. Tech - Computer Science and Engineering (MCSE)

Semester End Examination; Jan/Feb. - 2016

Probability and Statistics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. If A and B are any events, not necessarily mutually exclusive then prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$. 5

b. Define the following :
i) Probability
ii) Mutually exclusive event
iii) Independent events with one example for each. 6

c. What is the probability that a randomly chosen three letter sequence will not have any repeated letters? 5

d. A given lot of IC chips contain 2 percent defective chips. Each chip is tested before delivery. The tester itself is not totally reliable so that ;
P("Tester says chip is good" | Chip is actually good") = 0.95
OR
P('Tester says chip is defective" | "Chip is actually defective") = 0.94
If a tested device is indicated to be defective, what is the probability that it is actually defective? 4

2 a. Out of every 100 jobs received at a computer center, 50 are of class 1, 30 of class 2 and 20 of class 3. A sample of 30 jobs is taken with replacement :
i) Find the probability that the sample will contain ten jobs of each class.
ii) Find the probability that there will be exactly twelve jobs of class 2. 5

b. Define :
i) Discrete and continuous random variables with an example for each.
ii) Probability mass function of probability density function.
iii) Cumulative distribution of both random variables. 10

c. Using Hypergeometric pmf compute the probability of obtaining three defectives in a sample of size ten taken without replacement from a box of twenty components containing four defectives. 5

Contd…..2

UNIT - II

3 a. Consider a university computer center with an average rate of job submission λ = 0.1 jobs per second. Assuming that the number of arrivals per unit time is Poisson distributed, the interval time X, is exponentially distributed with parameterλ. What is the probability that an interval of 10 seconds elapses without job submission? 5

b. An analog signal received at a detector may be modeled as a Gaussian random variable N(200, 256) at a fixed point in time. What is the probability that the signal will exceed 240 microvolts? What is the probability that the signal is larger than 240 microvolts, given that it is larger than 210 microvolts? 10

c. Define Joint Distribution function. Also write its five properties. 5

4 a. Define :

i) Expectation ii) Variance 5

b. Prove that $Var[X+Y]=Var[X]+Var[Y]$; If X and Y are independent random variables. 10

c. State the central limit theorem. 5

UNIT - III

5 a. Define :

i) Conditional pmf and conditional pdf ii) Conditional expectation. 10

b. Consider the following program segment consisting of a repeat loop :

Repeat S until B

Let X_i denote the execution time for the time i^{th} iteration of statement group S. Assume that the sequence of tests of the Boolean expression B defines a sequence of Bernoulli trials with parameter P. The number N of iterations of the loop is a geometric random variable with parameter P so that $E[N]=1/P$. Let T denote the total execution time. Compute mean and variance. 10

6 a. Write a note on :

i) Random sums

ii) Stochastic Process

iii) Classification of stochastic process. 10

b. Given that n ≥ 1 arrivals have occurred in the interval (0, t), prove that the conditional joint pdf of the arrival times $T_1, T_2, ...T_n$ is 10

$$f\left[t_1, t_2, ...t_n \middle| N(t)=n\right]=\frac{n!}{t^n}, \quad 0 \le t_1, \le ...... \le t_n \le t$$

Contd…..3

UNIT - IV

7 a. Given a two state Markov chain with the transition probability matrix

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, \quad 0 \le a, \quad b \le 1, \quad \left\|1-a-b\right| < 1$$

Prove that the n-step Transition probability

Matrix $p(n) = P^n$ is 10

$$P(n) = \begin{bmatrix} \frac{b+a(1-a-b)^n}{a+b} & \frac{a-a(1-a-b)^n}{a+b} \\ \frac{b-b(1-a-b)^n}{a+b} & \frac{a+b(1-a-b)^n}{a+b} \end{bmatrix}$$

b. Write short notes on state classification and limiting distribution. 5

c. Consider a cascade of error-free binary communication channels. Write the transition probability matrix and the state diagram for this problem. 5

8 a. Determine the maximum call rate that can be supported by one telephone booth. Assume that the mean duration of a telephone conversation is three minutes and that no more than a three-minute (average) wait for the phone may be tolerated. What is the largest amount of incoming traffic that can be supported? 10

b. Explain open queing networks and closed queing networks. 10

UNIT - V

9 a. Define the terms :

i) Parameter estimation ii) Sampling distribution 10

iii) Efficiency iv) Consistency with usual notations.

b. Show that the sample variance S^2 is defined by

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$ is an unbiased estimator of the population variance σ^2. 10

10 a. Explain null hypothesis & alternate hypothesis by means of an example. 7

b. Prove that following :

i) $\hat{A}$ is an unbiased estimator of a; $E\left[\hat{A}\right] = a$

ii) $\hat{B}$ is an unbiased estimator of b; $E\left[\hat{B}\right] = b$ 9

iii) $Var\left[\hat{B}\right] = \frac{\sigma^2}{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)}$

c. Write a note on correlation analysis. 4

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