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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, M. Tech - Computer Science and Engineering (MCSE)

Semester End Examination; Jan - 2017

Probability and Statistics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Given $P(A) = 0.30,\ P(B) = 0.78,\ P(A \cap B) = 0.16$. Evaluate :
i) $P(A' \cap B')$ ii) $P(A' \cup B')$ iii) $P(A \cap B')$. 4

b. Show that, if event B is contained in event A then $P(B) \leq P(A)$. 6

c. How many even two digit numbers can be constructed out of the digits 3, 4, 5, 6 and 7 by considering that, i) no digit is repeated ii) repetition of digit is allowed. 6

d. A machine produces a total of 12000 bolts a day of which 3% are defective on an average. Calculate the probability that out of 600 bolts chosen at random, 12 will be defective. 4

2 a. Define random variable with an example. 6

b. Given a discrete random variable *X*, the event $A_X = \{s \varepsilon S \,|\, X(s) = x\}$. Show that the family of events $\{A_X\}$ defines an event space. 6

c. Suppose that a pair of fair dice are rolled and let the random variable *X* denote the sum of the points. Calculate :
i) $P(X \leq 10)$
ii) $P(5 < X \leq 10)$. 8

UNIT - II

3 a. Prove that the exponential distribution is memory less. 8

b. Suppose the life in weeks of a certain kind of computers has the probability density function,

$$f(x) = \begin{cases} 100/x^2 & x \geq 100 \\ 0, & x < 100 \end{cases}$$

What is the probability that none of three such computers will have to be replaced during the first 150 weeks of operations? 6

c. Give the properties of pdf and cdf of a continuous random variable. 6

4 a. Compute the mean and variance for,
i) Normal Distribution ii) Binomial distribution. 10

b. Show that, if $X_1(\mu_1,\sigma_1^2)$ and $X_2(\mu_1,\sigma_2^2)$ are independent random variables then the random variable $Y = X_1 + X_2$ is also normally distributed with mean $\mu_1 - \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$. 6

c. Consider an interactive system designed to handle a maximum of fifteen transactions per second. During the peak hour of its activity, transactions arrive at average rate of ten per second. Assuming that the number of transactions arriving per second follows a Poisson distribution, compute the probability that the system will be overloaded during the peak hour. 4

UNIT - III

5 a. What is a Stochastic process? How the Stochastic processes are classified? 10

b. A system requires 3 components of a certain type to function properly. All components are assumed to have a constant failure rate λ = 0.002. During a mission component *j* is put into operation for t_j time units and a component can fail only when it is in operation. The components 1, 2, and 3 have time of operation 1300, 1500 and 1200 respectively. Determine the number of spares needed in order to achieve the probability greater than 0.9. 10

6 a. You roll one red die and one green die. Define the random variables *X* and *Y* as follows,

X = The number showing on the red die

Y = The number of dice that show the number six. 8

Write down a table showing the joint probability mass function for *X* and *Y*, find the marginal distribution for *Y*, and computer *E(Y)*.

b. Describe the following Stochastic processes :

i) Markov Process

ii) Strictly Stationary Process 12

iii) Independent Process.

UNIT - IV

7 a. Show that the steady state probability P_n of a birth-death process being in state *n* is given by $P_n = \left(\frac{(\lambda_0, \lambda_1 .....\lambda_{n-1})}{(\mu_1, \mu_2 ....\mu_n)}\right) * P_0$, where P_0 is the probability being in the zero state, λ is the arrival rate and μ is the service rate. 12

b. A group of telephone subscribers is observed continuously during an 80 minute busy hour period. During this time they make 30 calls with the total conversation time being 4200 seconds. Compute the call arrival rate and traffic intensity. 8

8 a. Describe the significance of transition probability matrix of discrete parameter Markov chain. 10

b. With a neat diagram, explain the structure of closed queuing networks. 10

UNIT - V

9 a. The number of calls per hour to telephone trunk be Poisson distributed with parameter λ. Prove that maximum likelihood estimator of the average arrival rate is sample mean. 10

b. Execution times (in seconds) of 40 jobs processed by a computing center were measured and found to be :

10	19	90	40	15	11	32	17	4	152
23	13	36	101	2	14	2	23	34	15
27	1	57	17	3	30	50	4	62	48
9	11	20	13	38	54	46	12	5	26

10

Calculate the sample mean and sample variance. Find the 90 percent confidence intervals for the mean and variance of execution time of a job. Assume that the execution time is approximately normally distributed.

10 a. Illustrate least square method of linear regression. 8

b. Suppose the National Transportation Safety Board (NTSB) wants to examine the safety of compact cars, midsize cars, and full-size cars. It collects a sample of three for each of cars types. Using the hypothetical data provided below, test whether the mean pressure applied to the driver's head during a crash test is equal for each types of car using ANOVA.
Use α = 5%. Clearly state the null hypothesis used. 12

Compact cars	Midsize cars	Full –Size cars
643	469	484
655	456	427
702	525	402

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