U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, M. Tech - Mechanical Engineering (MMDN)

Semester End Examination; Jan - 2017

Finite Element Method

Time: 3 hrs *Max. Marks: 100*

***Note**: i) Answer **FIVE** full questions, selecting **ONE** full question from each unit.*
ii) Assume missing data, if any.

UNIT - I

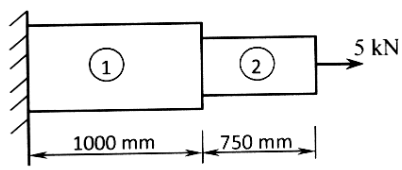
1 a. Discuss in brief the basic steps involved in FEM. 6

b. With necessary sketches, differentiate between essential and non-essential boundary conditions. 6

c. Derive an expression for potential energy functional of 3D elastic body is subjected to body force, surface force and point load components in its *x*, *y* and *z* directions. 8

2 a. Derive shape functions for a 2-noded bar element. 6

b. For the stepped bar shown in Fig. Q2(b), 14



$A_1 = 500mm^2; A_2 = 400mm^2$

$E_1 = 100GPa; E_2 = 200GPa$

Fig. Q 2(b)

Determine the following :

(i) Element stiffness matrices (ii) Nodal displacements (iii) B Matrices

(iv) Stresses in each elements (v) Support reaction.

UNIT - II

3 a. Derive shape functions for 3-noded triangular element. 8

b. With necessary sketches, explain the concept of ISO, Sub and Super parametric elements. 6

c. Obtain the Jacobean matrix for the triangular element shown in Fig. Q3(c). Also determine the area of triangular element. 6

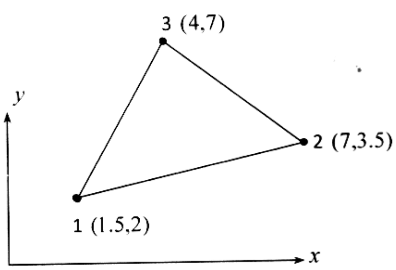
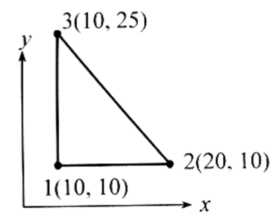


Fig. Q 3(c)

4 a. For the triangular element shown in Fig. Q4(a), the nodal displacement are given by,

$u_1 = 0.005\ mm; \quad u_2 = 0.00\ mm; \quad u_3 = 0.005\ mm;$

$v_1 = 0.002\ mm; \quad v_2 = 0.00\ mm; \quad v_3 = 0.00\ mm;$



All coordinates are in mm

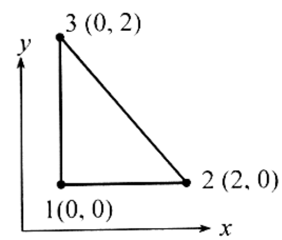
Fig. Q 4(a)

Determine the strain-displacement matrix, B and hence calculate element strain ε_x, ε_y, γ_{xy}. 10

b. Derive shape functions for 4-noded Tetrahedral element. 10

UNIT - III

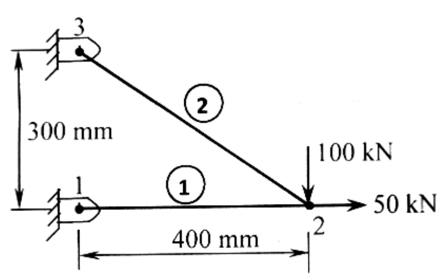
5. Derive strain-displacement matrix for an axi-symmetric triangular element and hence obtain strain-displacement matrix of axi-symmetric element shown in Fig. Q5. 20



All coordinates are in mm

Fig. Q 5

6. For the truss structure shown in Fig. Q6. Determine the nodal displacements, stress in member-1 and reaction at support 3. 20



$E=200\ GPa,\ A_1=1500\ mm^2,\ A_2=2000\ mm^2$

Fig. Q 6

UNIT - IV

7 a. Write Harmite shape functions of a 2-noded beam element and draw their variation along the element. 6

b. For the beam shown in Fig. Q 7(b), determine the nodal deflections, slops and the vertical deflection in the mid-point of distributed load. Use two element approximation and take E = 70 GPa, $I = 3x10^{-4}$ m^4. 14

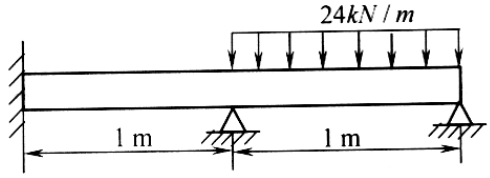


Fig. Q 7(b)

8 a. Write consistent mass matrix of plane truss and CST elements. 4

b. A one-dimensional bar of length L, modulus of elasticity E, mass density ρ and cross sectional area A is fixed at one end and free at other end. Determine its first two natural frequencies using two elements of equal length. 16

UNIT - V

9 a. Discuss the types of boundary conditions used in heat transfer problems. 6

b. Inner surface temperature of a composite wall shown in Fig. 9(b) is maintained at 20ºC. The convective heat transfer takes place at outer surface with h = 25 W/m^2 °C and T_∞ = -15ºC. Determine temperature distribution in the wall. 14

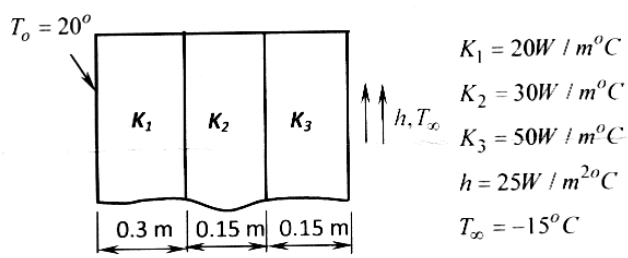


Fig. Q 9(b)

10. Fig.Q10 shows a uniform aluminum fin of diameter 20 mm. The root (left end) of the fin is maintained at a temperature of T_0=100ºC while convention takes place from the lateral (circular) surface and the right (flat) edge of the fin. Assuming K = 200 *W/m ºC*, h = 1000 W/m^2 *°C and* T_∞ = *20ºC*, determine the temperature distribution in the fin using a two-element idealization. 20

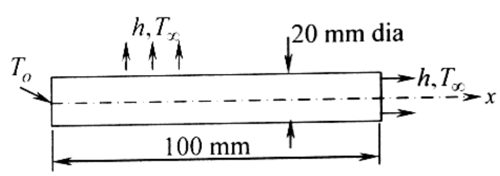


Fig. Q 10

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