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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, M. Tech – Mechanical Engineering (MMDN)

Semester End Examination; Jan/Feb - 2016

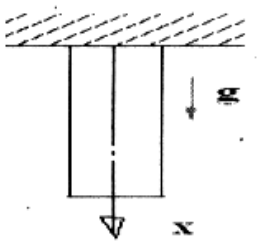
Theory of Elasticity

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each **unit**.*

UNIT - I

1 a. A one dimensional problem of prismatic bar loaded under its own weight can be modeled by the stress field $\sigma_x = \sigma_x(X), \sigma_y = \sigma_z = \tau_{xy} = \sigma_{yx} = \tau_{xz} = \tau_{zx} = \tau_{xy} = \tau_{yx} = 0$ with body forces $f_x = \rho g,\ f_y = f_z = 0.$ ρ is the mass density and g is the local acceleration of gravity. Using equations of equilibrium show that $\sigma_x = \rho g(1-x)$ 6



b. A body is subjected to three dimensional forces and state of stress in *xyz* frame at appoint in it is represented as,

$$\sigma_y = \begin{bmatrix} 200 & 200 & 200 \\ 200 & -100 & 200 \\ 200 & 200 & -100 \end{bmatrix} \ MPa$$ 8

Determine the normal, shear and resultant stress on :

i) The plane equally inclined to *xyz* planes ii) Octahedral planes.

c. A point in a machine member is subjected to pure shear stress of 45 MPa. Draw Mohr circle and find normal stresses on planes whose normal makes 45º and 135º with *x* axis. 6

2 a. Derive the equilibrium equations for a 2D state of stress including body loads and hence prove the equality of cross shears. 12

b. For the given stress, determine the principal stresses and their directions,

$$\sigma_y = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$ 8

UNIT - II

3 a. Derive strain displacement relations for 2D. 10

Contd…..2

b. Strain components are given by $\varepsilon_x = 0.1$, $\varepsilon_y = 0.2$, $\varepsilon_z = 0.3$, $\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0.16$. Determine the strain components in new coordinate system, which is oriented by an angle $xx' = \frac{\pi}{4}$, $yy' = \frac{\pi}{4}$, $zz' = 0$. 10

4 a. "The strain must be compatible" What is the physical interpretation of this statement? Derive the compatibility equations for 3D. 12

b. The displacement field for a body is given by $u = \left(x^2 + y\right)i + \left(3 + z\right)j + \left(x^2 + 2y\right)k$. Determine the strain components at (3, 1, -2) and deformed position of point originally at (3, 1, -2). What is the strain in direction l = m = n = 1/√3. 8

UNIT - III

5 a. Explain plane stress and plane strain problem. Give examples and write stiffness and compliance matrices. 10

b. The stress tensor at a point in MPa, is given by,

$$\sigma_y = \begin{bmatrix} 210 & 160 & 120 \\ 160 & -240 & 100 \\ -120 & 100 & 150 \end{bmatrix}$$ 6

Determine the strain tensor and Lame's constants E = 210 GPa, and v = 0.3.

c. State Saint Venant's principle. Explain its importance in theory of elasticity. 4

6 a. Explain methods of solution to elasticity problems. 6

b. Write short notes on : 14

i) Principle of super position ii) Reciprocal theorem.

UNIT - IV

7 a. What is stress function? Derive the Biharmonic equation in Cartesian coordinates for a two dimensional plane stress in the absence of body force. 8

b. Show that $\sigma_{xx} = \left(-Pxy\right)/I$, $\sigma_{yy} = 0$, $\tau_{xy} = \left(P/2I\right)\left(d^2/4 - y^2\right)$ are expressions for stress components in solving a problem for a narrow cantilever beam under an edge load P, span L and of rectangular cross section h x d. 12

8 a. Using Saint Venant's semi inverse approach, derive the necessary governing equations and boundary equations for the complete solution of torsion of prismatic bar. 14

b. A hollow Aluminium tube of rectangular cross section shown in Fig. 8b is subjected to a torque of 56,500 N-m along its longitudinal axis. Determine the shearing stresses and the angle of twist. G = 27.6 GPa. 6

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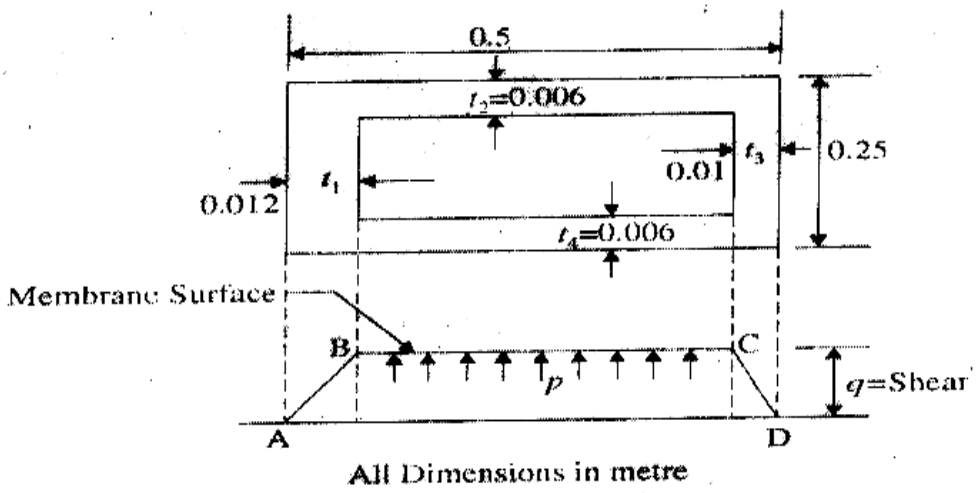

Figure 8b

UNIT - V

9 a. Derive the expression for radial and tangential stresses for a thick cylinder subjected to internal and external pressure. 12

b. A hollow cylinder of 120 mm outside diameter and 80 mm inside diameter is subjected to internal pressure of 120 MPa and external pressure of 40 MPa. Determine the tangential stresses at inner and outer diameter and the radial and tangential stresses at the mean diameter. 8

10 a. Discuss the thermo elastic stress strain relation. 6

b. Derive the expressions for radial and tangential stresses in a thin hollow disk of uniform thickness subjected to uniform temperature distribution. 14

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