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THE SPLITE QUITABLE DOMIATION NUMBER OF A GRAPH

Puttaswamy[1], K. B. Murthy[2]

1Department of Mathematics, PES college of Engineering, Mandya-571401, India
2Department of Agril. stats and Applied Mathematics, UAS College of Agriculture, GKVK Campus, Bangalore-65

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Corresponding Author
Puttaswamy
Department of Mathematics, PES college of Engineering, Mandya-571401, India
prof.puttaswamy@gmail.com

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ABSTRACT

An equitable dominating set D of a graph G = (V, E) is a split equitable dominating set if the induced subgraph hV − Di is a disconnected. The split equitable domination number ϒse(G) of a graph G is the minimum cardinality of a split equitable dominating set. In this paper, we initiate the study of this new parameter and present some bounds and some exact values for ϒse(G). Also Nordhaus–Gaddum type results are obtained.

Mathematics Subject Classification : 05C69.

INTRODUCTION

In this paper we assume G = (V, E) is an undirected simple graph with finite vertex set V and edge set E. Our notation generally follows that used in [1]. A set D of vertices in a graph G = (V,E) is called a dominating set of G if every vertex in V − D is adjacent to some vertex in D. The domination number ϒ(G) of a graph G is the minimum cardinality of a dominating set of G. For more detail see [2]. A subset D of V is called an equitable dominating set if for every $v \in V - D$ there exists a vertex u 2 D such that *uv* ϵE(G) and | deg(*u*) − deg(*v*) |≤ 1.

The Minimum cardinality of such a dominating set is denoted by ϒe(G) and is called the equitable dominating number of G, see [4]. A dominating set D of a graph G is a split dominating set if the induced subgraph (V − D) is disconnected. The split dominating number ϒs(G) of a graph G is the minimum cardinality of a split dominating set of G. This concept was introduced by Kulli and Janakiram [3]. Analogously in this paper we now define the following concept. An equitable dominating set D of a graph G is a split equitable dominating set if the induced subgraph (V − D) is disconeted. The split equitable dominating number se(G) of a graph G is the minimum cardinality of a split equitable dominating set. We note that se-set exists if the graph is not complete and either contains a non-complete component or contains atleast two non-trivial components se(G) not exist, if the graph totally equitable disconnected (All the vertices of G are equitable isolated) like Sn. We also note that se-set not exists if the graph G is totally disconnected.

A vertex $u \in V$ is said to be degree equitable adjacent with a vertex $v \in V$ if u and v are adjacent and split $|\deg(u) - \deg(v)| \leq 1$.

The split equitable dominating set D is said to be a minimal equitable dominating set if no proper subset of D is split equitable dominating set. Similarly, as the standard dominating set every minimum equidominating set is minimal but the converse not true some good example
to show the different.

If a vertex $u \in V$ be such that $|\deg(u) - \deg(v)| \geq 2$ for all $v \in N(u)$, then u is in every equitable dominating set such vertices are called equitable isolates.

Let $u \in V$. The equitable neighbourhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V\ ;\ v \in N(u), |\deg(u) - \deg(v)| \leq 1\}$. The cardinality of $N_e(u)$ is denoted by $deg_e G(u)$. The maximum and minimum equitable degree of a vertex in G are denoted respectively by $\Delta_e(G)$ and $\delta_e(G)$.

Proposition 1 : *For any graph G,* (*i*) $\gamma_e(G) \leq \gamma_{se}(G)$ (*ii*) $\gamma_s(G) \leq \gamma_{se}(G)$.

Proof : Let D be the minimum split equitable dominating set of G. Now, since D is a split equitable dominating set then D is equitable dominating set. Hence $\gamma_e(G) \leq |D| = \gamma_{se}(G)$. In the following Proposition, We obtain the exact value of split equitable domination number for some standard graphs.

The proof of (ii) is similar.

Proposition 2 : (*i*) $\gamma_{se}(C_p) = \left\lceil \frac{p}{3} \right\rceil$, *if* $p \geq 4$.
(*ii*) $\gamma_{se}(K_{m,n}) = m$, *if* $2 \leq m \leq n$ *and* $\mid m - n \mid \leq 1$.
(*iii*) $\gamma_{se}(W_p) = 1 + \left\lceil \frac{p-1}{3} \right\rceil$, *if* $p \geq 5$.
(*iv*) $\gamma_{se}(P_p) = \left\lceil \frac{p}{3} \right\rceil$, *if* $p \geq 3$.

A vertex v is said to be equitable and vertex if $deg_e(v) = 1$.

Theorem 3 : *For any graph* G, $\gamma_{se}(G) \leq \alpha_e(G)$, *where* $\alpha_e(G)$ *is the equitable covering number of* G.

Proof : Let D be the maximum equitable independent set of vertices in G. i.e., $|D| = \beta_e(G)$ .Then D has atleast two vertices and every vertex in D is equitable adjacent to some vertex in $V - D$. This implies that $V - D$ is a split equitable dominating set of G.

$$\begin{aligned} \text{Hence } \gamma_{se}(G) &\leq |V - D| \\ &= p - \beta_e(G) \\ &= \alpha_e(G). \end{aligned}$$

Corollary 3.1 : *For any graph* G, $\gamma_{se}(G) + \beta_e(G) \leq p$. *where* $\beta_e(G)$ *is the equitable independence number of* G.

Theorem 4 : $\alpha_e(G) + \beta_e(G) \leq p$, *where* $\alpha_e(G)$ *and* $\beta_e(G)$ *are the equitable covering number and equitable independence number of* G.

Theorem 4.1 : *For any graph* G, $\gamma_e(G) + \gamma_{se}(G) \leq p$.

Proof: Since $\gamma_e(G) \leq \beta_e(G)$ and $\gamma_{se}(G) \leq \alpha_e(G)$

$$\begin{aligned} \gamma_e(G) + \gamma_{se}(G) &\leq \alpha_e(G) + \beta_e(G) \\ &= p. \end{aligned}$$

Thus, $\gamma_e(G) + \gamma_{se}(G) \leq p$.

Theorem 5 : *For any graph* G, $\gamma_{ie}(G) + \gamma_{se}(G) \leq p$.

Proof : Since $\gamma_{ie}(G) \leq \beta_e(G)$ and $\gamma_{se}(G) \leq \alpha_e(G)$ $\gamma_{ie}(G) + \gamma_{se}(G) \leq \beta_e(G) + \alpha_e(G) = p$.
Thus, $\gamma_{ie}(G) + \gamma_{se}(G) \leq p$.

The sharpness of this inequality can be seen with the path P_4 which has $\gamma_{ie} = \gamma_{se} = 2$.

Hence $\gamma_{ie}(P_4) + \gamma_{se}(P_4) = 4 = p$.

Theorem 6 : *A split equitable dominating set* D *of* G *is minimal for each vertex* $v \in D$, *one of the following three condition holds :*
(*i*) *there exists a vertex* $u \in V - D$, *such that* $N_e(u) \bigcap D = \{v\}$.
(*ii*) v *is an equitable isolated vertex in* $\langle D \rangle$.
(*iii*) $\langle (V - D) \bigcup \{v\} \rangle$ *is connected.*

Proof : Suppose D is minimal and there exists a vertex $v \in D$ such that v does not hold any of the above conditions. Then by condition (*i*) and (*ii*), $D_1 = D - \{v\}$ is a equitable dominating set of G. Also by (*iii*), $\langle V - D \rangle$ is disconnected. This implies that D_1 is a split equitable dominating set of G, a contradiction.

Theorem 7 : *If* G *is regular or bi-regular graph with atleast one equitable end vertex, then* $\gamma(G) = \gamma_s(G) = \gamma_{se}(G) = \gamma_e(G)$.

Proof : It is clear that, if G is regular or bi-regular then any two vertices are equitable adjacent. Hence $\gamma(G) = \gamma_e(G)$.
Now, let v be an equitable end vertex in G and let w be the vertex which is equitable adjacent to v and D be the γ_e - set of G. We consider the following cases:

***Case* I :** If $w \in D$, then D is also split equitable dominating set and hence $\gamma_{se}(G) \leq \gamma_e(G)$ and by observation $\gamma_e(G) \leq \gamma_{se}(G)$. Thus $\gamma_e(G) = \gamma_{se}(G)$.

***Case* II :** If $w \in V - D$, then $v \in D$ and hence $(D - \{v\}) \bigcup \{w\}$ is a

γ_{se} - set of G. Thus $\gamma_{se}(G) = \gamma_e(G)$.

Theorem 8 : *For any graph G,* $\gamma_{se}(G) \leq \frac{p\Delta_e}{(\Delta_e + 1)}$.

Proof : Let D be a minimum split equitable dominating set of G. Since D is minimal, by **Theorem 6** for each vertex $v \in D$ there exists a vertex $u \in V - D$ such that v is equitable adjacent to u. This implies that $V - D$ is an equitable dominating set of G. Thus $\gamma_e(G) \leq |\ V - D\ |$

$= p - \gamma_{se}(G) \ldots\ldots (1)$

Further, it is known that

$\gamma_e(G) \geq \frac{p}{\Delta_e + 1} \ldots\ldots (2)$

From (1) and (2), we have, $\gamma_{se}(G) \leq \frac{p\Delta_e}{(\Delta_e + 1)}$

Theorem 9 : *For any graph P_p with $p \geq 5$ vertices,* $\gamma_{se}\ (\overline{P_p}) \leq p - 3$.

Proof : From the definition of the complete graph, it is clear that P_p is bi-regular graph with degree $p - 2$ or $p - 3$, then the split equitable dominating exists for $\overline{P_p}$ only if $p \geq 4$. If $p = 4$ then $\overline{P_p} \simeq P_p$, hence $\gamma_{se}(P_4) = 2$. When $p \geq 5$, let $D = \{v_4, v_5, \ldots, v_p\}$.
Then $D \neq \emptyset$ and $V - D = \{v_1, v_2, v_3\}$.
$V(\overline{P_p}) - D = \{v_1, v_2, v_3\}$, it is clear that all the vertices in $V(\overline{P_p}) - D$ are equitable dominating set of G, and also v_2 is isolated vertex in $\langle V(\overline{P_p}) - D \rangle$ that means $\langle V(\overline{P_p}) - D \rangle$ is disconnected. Hence D is split equitable dominating set of $\overline{P_p}$. Therefore $\gamma_{se}\ (\overline{P_p}) \leq p - 3$, when $p \geq 5$, we obtain the Nordhus–Gaddum type result.

Theorem 10 : $\gamma_{se}(P_p) + \gamma_{se}\ (\overline{P_p}) \leq \left\lceil \frac{p}{3} \right\rceil + p - 3,\ p \geq 5.$

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