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COMPLEMENTARY PENDANT DOMINATION IN GRAPHS

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Abstract

Let G be any graph. A dominating set S in G is called a complementary pendant dominating set if $\langle V - S\rangle$ contains at least one pendant vertex. The minimum cardinality of a complementary pendant dominating set is called the complementary pendant domination number of G, denoted by $\gamma_{cpd}(G)$. In this paper we begin to study this parameter. We calculate exact values of γ_{cpd} for some families of standard graphs. Further the bounds and the relation with other domination parameters are estimated for γ_{cpd}.

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Key Words and Phrases: Pendant Dominating set, Complementary Pendant Domination Number.

1 Introduction

Let G be any graph. The concept of paired domination is an interesting concept introduced by T. W. Haynes in with the following application in mind. If we think of each vertex v as the possible location for a guard capable of protecting each vertex in its closed neighborhood, then *domination* requires every vertex to be protected. For total domination, each guard must, in turn, be protected by other guard. But for paired-domination, each guard is assigned another adjacent one, and they are designated as backups for each other. Motivated by this the concept, pendant domination was introduced and studied in [3]. In this paper, we introduce and study the complementary pendant in graphs. For any graph theoretic terms not defined here, refereed to [1, 2].

2 Basic Definition and Notation

For each vertex $v \in V$, the open neighborhood of v is the set $N(v)$ containing all the vertices u adjacent to v and the closed neighborhood of v is the set $N[v]$ containing v and all the vertices u adjacent to v. A vertex of a degree one is called a pendant vertex. A subset S of $V(G)$ is a dominating set if $N[S] = v$. The least cardinality of a dominating set is called the domination number, denoted by $\gamma(G)$.

A dominating set S of a graph G is said to be paired dominating set of G if the $\langle S\rangle$ contains at least one perfect matching. Any paired dominating set with minimum cardinality is called a minimum paired dominating set. The cardinality of the minimum paired dominating is called the paired domination number of G and is denoted by $\gamma_{pd}(G)$. A dominating set S is called a total dominating set if $\langle S\rangle$ contains no isolated vertex. The cardinality of the minimum total dominating set is called the total domination number of G and is denoted by $\gamma_t(G)$. A dominating set S in G is called a pendant dominating set if $\langle S\rangle$ contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number denoted by $\gamma_{pe}(G)$.

The corona of two disjoint graphs G_1 and G_2 is defined to be the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the ith vertex of G_1 is adjacent to every vertex in the ith copy of G_2. If G and H are disjoint graphs,then the join of G and H denoted by $G \vee H$ is the graph such that $V(G \vee H) = V(G) \cup V(H)$ and $E(G \vee H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$.

3 The Complementary Pendant Domination Number of a Graph

A dominating set S in G is called a complementary pendant dominating set if $\langle V - S \rangle$ contains at least one pendant vertex. The minimum cardinality of a complementary pendant dominating set is called the complementary pendant domination number of G, denoted by $\gamma_{cpd}(G)$.

Example 1. *For the graph G in the Figure 1.1, one can verify that $S = \{3, 4, 9, 12, 15\}$ is a dominating set with $\{1, 2\}$ as a pendant vertex in $< V - S >$. Hence S is a $\gamma_{cpd}-$ set. Further $\{3, 4, 10, 15\}$ is the minimum $\gamma_{cpd}-$ set and so $\gamma_{cpd}(G) = 4$.*

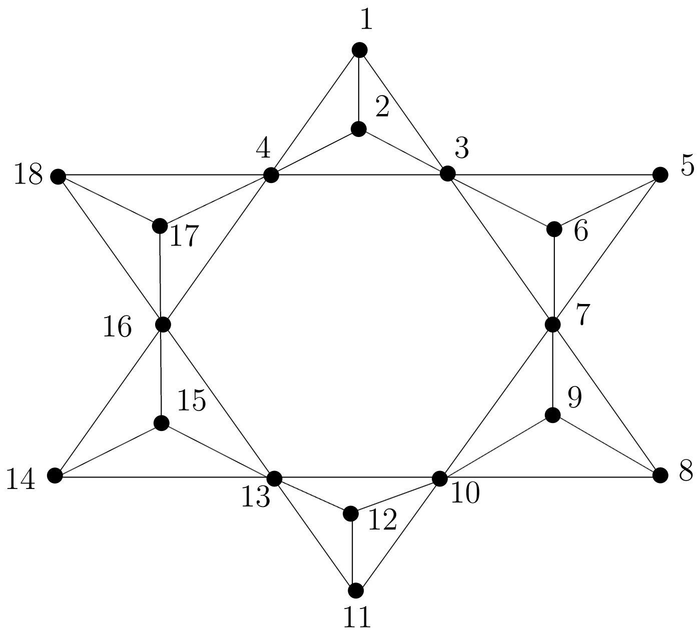

Figure 1.1

From the definition of the complementary pendant domination number, we have the following observation:

Observation 1. *Let G be totally disconnected or G a star. Then complementary pendant domination number is not defined for G. Hence throughout this article by a graph G we assume $m \geq 1$.*

- $\gamma(P_n) = \gamma_{pe}(P_n) = \Gamma(P_n) = \gamma_{cpd}(P_n)$ if and only if $n = 4$.

- $\gamma(K_{m,n}) = \gamma_{pe}(K_{m,n}) = \Gamma(K_{m,n}) = \Gamma_{pe}(K_{m,n}) = \gamma_{cpd}(K_{m,n})$ if and only if $m, n = 2$.
- Let G be bistar, then $\Gamma(G) = \gamma_{cpd}(G)$.

When the new domination parameter is defined, it is natural to find the relation between the new parameters and the existing parameter. In the following examples, we note observe that there is no relation between γ_t, γ_{pr} and γ_c. First let G be a cycle C_5, then γ_{cpd} does not exceed any of these parameters. Next, suppose $G \cong P_4 \circ \overline{K}_4$, then we will have $\gamma_t(G) = \gamma_{pr}(G) = \gamma_c(G) = 4$ but $\gamma_{cpd}(G) = 10$. Hence, there is no relation between theses parameters.

Lemma 1. *The following are true*
(i) $\gamma_{cpd}(K_n) = n - 2, n \geq 3$.
(ii) $\gamma_{cpd}(K_{m,n}) = m + n - 3, m, n \geq 2$.
(iii) $\gamma_{cpd}(P_n) = \gamma_{cpd}(C_n) = \lceil \frac{n}{3} \rceil, n \geq 4$.
(iv) $\gamma_{cpd}(W_n) = 2, n \geq 4$.
(v) $\gamma_{cpd}(\overline{C_n}) = \gamma_{cpd}(\overline{P_n}) = n - 3$, $n \geq 3$.
(vi) *For any graph* G, $\gamma(G) \leq \gamma_{cpd}(G)$.

Proof. **(i)** Every induced subgraph of K_n is complete. For any two adjacent vertices $\{u, v\}$ in K_n, the set $S = \{V(K_n) - \{u, v\}\}$ will be a complementary pendant dominating set of K_n. Hence $\gamma_{cpd} = |S| = n - 2$.

(ii) Let $\{V_1, V_2\}$ are two partite set in $K_{m,n}$ Choose an arbitrary path $P_3 = \{v_1, v_2, v_3\}$ in $K_{m,n}$. Then, the set $S = V - \{v_1, v_2, v_3\}$ will be a complementary pendant dominating set of $K_{m,n}$. Hence, $\gamma_{cpd}(K_{m,n}) \leq\mid S \mid= m + n - 3$. On the other hand, it may be noted that any subset S' of size at least $m + n - 4$, the set $V - S$ has minimum degree at least 2. Thus, we must have $\gamma_{cpd}(K_{m,n}) \geq m + n - 3$, proving (ii).

(iii) Let G be a Cycle or a Path with $n \geq 4$ vertices. Then $S = \{v_1, v_4, v_7, \ldots, v_n\}$ will be a $\gamma-$ set of G and $< V - S >$ contains a pendant vertex and so S itself a complementary pendant dominating set of G. Therefore $\gamma_{cpd}(G) =\mid S \mid= \lceil \frac{n}{3} \rceil$.

(iv) Let W_n be a wheel with $n \geq$ vertices. and let u be a vertex at the center of W_n. Clearly $\{u\}$ will be a dominating set but $V - \{u\}$ is a cycle C_{n-1}. Hence $\gamma_{cpd}(W_n) \geq 2$. Next, choosing an arbitrary vertex v on cycle C_{n-1}, the set $S = \{u, v\}$ will be a minimum complementary pendant dominating set. Therefore $\gamma_{cpd}(W_n) = 2$.

(v) Clearly $\delta(\overline{C_n}) = n - 3$ and so there exist a vertex v_1 in $\overline{C}_n$ which is not adjacent to two vertices v_2 and v_n. Now the set $S = V - \{v_1, v_2, v_n\}$ is a complementary pendant dominating set. So $\gamma(\overline{C_n}) \leq |V - \{v_1, v_2, v_n\}| = n - 3$. Therefore $\gamma_{cpd}(\overline{C_n}) = n - 3$.

(vi) Since every complementary pendant dominating set is also a dominating set of a graph G, it follows that $\gamma(G) \leq \gamma_{cpd}(G)$. □

Proposition 1. *Let G be a n-pan Graph, of order $n \geq 4$. Then $\gamma_{cpd}(G) = \lceil \frac{n}{3} \rceil$.*

Theorem 1. *Let $G \cong K_m(a_1, a_2, \ldots, a_m)$ be a multistar graph, with $a_1 \leq a_2 \leq a_3 \leq \ldots \leq a_n$. Then $\gamma_{cpd}(G) = |a_1| + |a_2| + m - 2$.*

Proof. Let $G \cong K_m(a_1, a_2, \ldots, a_m)$ be a multi star of order $a_1 + a_2 + \cdots + a_m + m$. Assume that $a_1 \leq a_2 \leq \cdots \leq a_m$. Let u and v be a two adjacent supported vertices of G. The set S contains leaves of u, v and all the supported vertices of a multistar graph G except u,v. Therefore $\gamma_{cpd}(G) =| S |= |a_1| + |a_2| + m - 2$. □

Proposition 2. *Let G be a bistar graph then $\gamma_{cpd}(G) = m+n$.*

Proof. Let G be a bistar graph. The set $S = \{u, v\}$ is the dominating set of G. Then $< V - S >$ contains a pendant vertex, therefore S is a complementary pendant dominating set of G. So $\gamma_{cpd}(G) = |S| = \{m + n + 2\} - 2 = m + n$. □

Theorem 2. *Let G be a Barbell graph. Then $\gamma_{cpd}(G) = 2(n - 1)$.*

Proof. Let G be a barbell graph and let $V(G) = \{v_1, v_2, \ldots, v_{2n}\}$. Let v_1 and v_2 be the adjacent vertices of G attached to the copies of complete graph. Then, clearly the set $S = \{v_1, v_2\}$ is a dominating set of G. Now the set $S' = V - S$ is a complementary pendant dominating set of G. Therefore $\gamma_{cpd}(G) = 2n - 2 = 2(n - 1)$. □

Theorem 3. *Let G be a Ladder graph. Then $\gamma_{cpd}(G) = \lceil \frac{n}{2} \rceil + 2$.*

Proof. Let G be a ladder graph, fix an edge u_2v_2 of G. Then for any $\gamma-$ set S of $P_2 \times P_{n-3}$, the set $S = S' \cup \{u_2, v_2\}$ be the minimum complementary pendant dominating set of G. Hence $\gamma_{cpd}(P_2 \times P_n) = \gamma(P_2 \times P_{n-3}) + 2 = \lceil \frac{n}{2} \rceil + 2$. □

Theorem 4. *If T is a tree of order $n \geq 3$, then $\Delta(T) \leq \gamma_{cpd}(T)$. Furthermore $\gamma_{cpd}(T) = \Delta(T)$ if and only if T is a wounded spider graph which is not a star.*

Proposition 3. *For $r \geq 2$, if G is a $r-$regular graph. Then $\gamma_{cpd}(G) \leq \gamma(G) + r - 2$.*

Theorem 5. *Let G be a graph with n vertices. Then $\gamma(G) + \gamma_{cpd}(G) \leq n$.*

Proof. Let S be a complementary pendant dominating set. Then S is a dominating set and $< V - S >$ contains a pendant vertex. obviously, $\gamma_{cpd}(G) \leq |S|$. Since S is a dominating $< V - S >$ is also a dominating. Thus $\gamma(G) \leq \mid V - S \mid$. Hence $\gamma(G) + \gamma_{cpd}(G) \leq \mid S \mid + \mid V - S \mid = n$, proving the result. □

Proposition 4. *Let G be any graph. Then $\lceil \frac{n}{1+\Delta(G)} \rceil \leq \gamma_{cpd}(G) \leq n - \Delta(G)$.*

Theorem 6. *Let G be any graph with n vertices. Then,*

$$\gamma_{cpd}(G \vee P_m) = \begin{cases} min\{m, n\}, & \text{if } G \text{ contains a pendant vertex.} \\ n, & \text{otherwise.} \end{cases}$$

Proof. Let G be any graph of order n and let P_m be a path of order m. Let $\{v_1, v_2, \ldots v_n\}$ and $\{u_1, u_2, \ldots u_m\}$ are vertices of G and P_m respectively. If G contains a pendant vertex then clearly $\gamma_{cpd}(G \vee P_m) = min\{m, n\}$. Then $< V(G \vee P_m) - \{v_1, v_2, \ldots v_n\} >$ contains a pendant vertex. Therefore $\gamma_{cpd}(G \vee H) = \mid G \mid = n$. □

Proposition 5. *If G is a graph, then $\gamma_{cpd}(G) = 1$ if $G \cong T \vee K_1$, where T is a tree.*

Proof. Assume $G \cong T + K_1$, then the set $\{v\}$ is a complementary pendant dominating set of G. where $V(K_1) = \{v\}$. Conversely if $\gamma_{cpd}(G) = 1$, then there exist a complementary pendant dominating set S of G with $\mid S \mid = 1$. Such that $< V(G) - S >$ is tree, since each vertex in $< V(G) - S >$ is adjacent to the vertex S. $G \cong T \vee K_1$

where $T =< V(G) - S >$. □

Theorem 7. *Let G be a connected graph with n vertices. Then $\gamma_{cpd}(G \circ K_1) = n$.*

Proof. Let us choose u and v be any two leaves of adjacent supported vertices of the graph $(G \circ H)$. The set $S = \{V - N(u, v)\} \cup \{u, v\}$ will be a complementary pendant dominating set of $(G \circ H)$. Then $|S| \leq |V - N(u, v)| \cup \{u, v\} = n - 2 + 2 = n$. □

Theorem 8. *Let T_1 and T_2 be any two trees of order n_1 and n_2 respectively. Then $\gamma_{cpd}(T_1 \circ T_2) = n_1$.*

Proof. Let $V(T_1)$ denotes the vertex set of T_1 and $V(T_1)$ is a dominating set of $T_1 \circ T_2$. Then $< V(T_1 \circ T_2) - V(T_1) >$ contains a pendant vertex, therefore $V(T_1)$ is a complementary pendant dominating set of $T_1 \circ T_2$. $\gamma_{cpd}(T_1 \circ T_2) =| V(T_1) |= n_1$. □

Theorem 9. *Let G be any graph, if $diam(G) \geq 3$ and G contains a no isolated vertex. Then $\gamma_{cpd}(\overline{G}) = 2$.*

Proof. Let u and v be two vertices of G such that $d(u, v) = diam(G) \geq 3$. Obviously u and v dominates $\overline{G}$ since there is no vertex in G adjacent to both u and v. Hence $\{u, v\}$ dominates $\overline{G}$ and $\gamma_{cpd}(\overline{G}) \leq 2$. If $\gamma_{cpd}(\overline{G}) = 1$, then G has an isolated vertex,contrary to the hypothesis. □

Proposition 6. *Let G be a triangle free graph of order at least 4. Then $\gamma_{cpd}(\overline{G})$ = 2 or 3.*

Proof. Let G be a triangle free graph. If G contains an isolated vertex then clearly $\gamma_{cpd}(\overline{G}) = 2$. Suppose G has no isolated vertex, then G contain at least one edge say $e = uv$. As G is triangle free no vertex in G can be adjacent to both u and v. Thus $S = \{u, v\}$ will be a $\gamma-$set in $\overline{G}$. Now, for any vertex $w \in V(G)$, the set $S \cup \{w\}$ will be a γ_{cpd}-set in $\overline{G}$. Hence, $\gamma_{cpd}(\overline{G}) = 3$. □

References

[1] J.A.Bondy, U.S.R Murty, *Graph theory with application*, Elsevier science Publishing Co, Sixth printing, 1984.

[2] T.W.Haynes, S.T.Hedetniemi, P.J.Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, New york, 1998.

[3] Nayaka S.R, Puttaswamy and Purushothama S. *Pendant Domination in Graphs.* In communication.