



## P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

**First Semester, Mechanical Engineering (MMDN)**

**Make-up Examination; Feb - 2017**

**Finite Element Methods**

Time: 3 hrs

Max. Marks: 100

**Note:** i) Answer any **FIVE** full questions, selecting at least **ONE** from each unit.

ii) Missing data if any maybe suitably assume

### UNIT-I

- |      |   |    |
|------|---|----|
| 1 a. | Differentiate between essential and non-essential boundary conditions.  | 6  |
|      | b. Explain convergence criteria of a displacement model and mention how they are satisfied by the selected displacement function.   | 6  |
|      | c. Derive an expression for potential energy functional for a 3D elastic body subjected to body force, surface force and point load components in its x,y and z directions. | 8  |
| 2.a  | Derive shape functions for a 3-noded bar element in natural coordinates and plot their variations along the element   | 6  |
| b.   | For the stepped bar shown in Fig Q 2(b), determine nodal displacements, element stresses and support reaction. Take E=200GPa.   | 14 |

### UNIT-II

- |      |   |    |
|------|---|----|
| 3.a  | Derive strain displacement matrix, B for a CST element.   | 6  |
| b.   | With necessary sketches, explain the concept of Iso, Sub and Super parametric elements.   | 6  |
| c.   | The nodal coordinates of a triangular element at node 1, 2 and 3 are (1,1), (4,1) and(1,5) respectively. The nodal displacement are given by<br>$u_1 = 0.005mm; \quad u_2 = 0.0mm; \quad u_3 = 0.005mm$<br>$v_1 = 0.002mm; \quad v_2 = 0.0mm; \quad v_3 = 0.0mm$<br>Determine the strain-displacement matrix, B and hence calculate element strains $\epsilon_x, \epsilon_y, \gamma_{xy}$ | 8  |
| 4 a. | Obtain the Jacobian matrix for the quadrilateral element  | 10 |
| b    | Derive shape functions for 4-noded Tetrahedral element.   | 10 |

### UNIT-III

- |      |   |    |
|------|---|----|
| 5 a. | Derive the shape functions for an axi-symmetric triangular element.   | 8  |
| b.   | The element of an axi symmetric body is rotating with a constant speed of 200 rpm and subjected to an external pressure of 2 MPa as shown in Fig Q 5(b). If the mass density of the material $\rho = 7.6 \times 10^{-6} \text{ kg/mm}^2$ , evaluate the equivalent force at nodes. The nodal coordinates shown in figure in mm. | 12 |

6. For the truss structure shown in Fig Q 6, determine the nodal displacements, stress in horizontal member and reaction at top support. 20

**UNIT-IV**

- 7.a Write Hermite shape functions of a 2-noded beam element and draw their variations along the element. 6
- b. For the beam shown in Fig Q 7(b), determine the nodal deflections, slopes and the vertical deflections at the mid-point of distributed load. Use two element approximation and take  $E = 70 \text{ GPa}$ ,  $I = 3 \times 10^{-4} \text{ m}^4$ . 14
- 8.a Write consistent mass matrix of plane truss element. 2
- b. A one-dimensional bar of length  $L$ , modulus of elasticity  $E$ , mass density  $\rho$  and cross-sectional area  $A$  is fixed at one end and free at other end. Determine its first two natural frequencies using two elements of equal length. 18

**UNIT-V**

- 9 a. Using Galerkin's approach, derive the element conduction matrix for 1D element used for steady state heat transfer problems. 8
- b. Consider a brick wall (Fig Q 9(b)) of thickness  $L = 0.3 \text{ m}$ ,  $K = 0.7 \text{ W/m}^\circ\text{C}$ . The inner surface is at  $28^\circ\text{C}$  and the outer surface is exposed to cold air at  $-15^\circ\text{C}$ . The heat transfer coefficient associated with the outside surface is  $h = 40 \text{ W/m}^2\text{C}$ . Determine the steady-state temperature distribution within the wall and also the heat flux through the wall. Use two-element model. 12
10. Fig Q 10 shows a uniform aluminium fin of diameter 20 mm. The root (left end) of the fin is maintained at a temperature of  $T_0 = 100^\circ\text{C}$  while convection takes place from the lateral (circular) surface and the right (flat) edge of the fin. Assuming  $K = 200 \text{ W/m}^\circ\text{C}$ ,  $h = 1000 \text{ W/m}^2\text{C}$  and  $T_\infty = 20^\circ\text{C}$ , determine the temperature distribution in the fin using a two-element idealization. 20

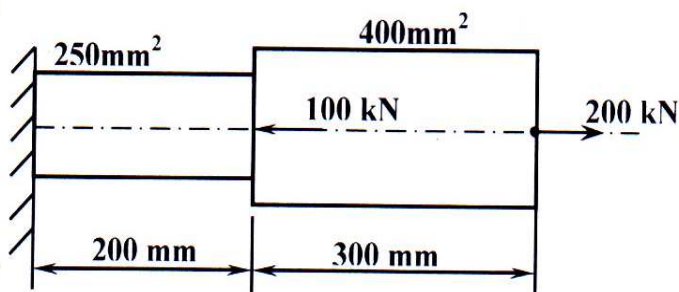


Fig. Q 2(b)

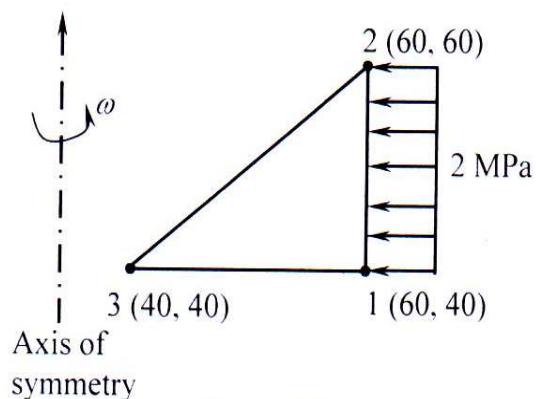


Fig. Q 5(b)

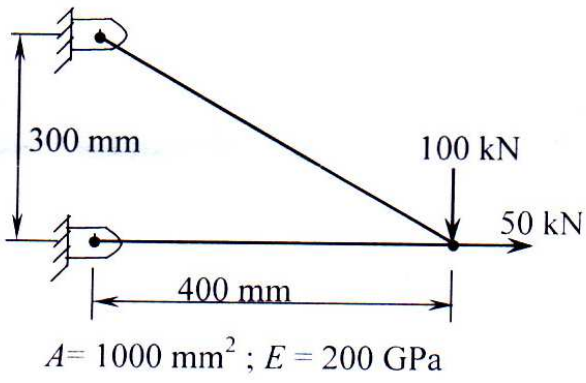


Fig. Q 6

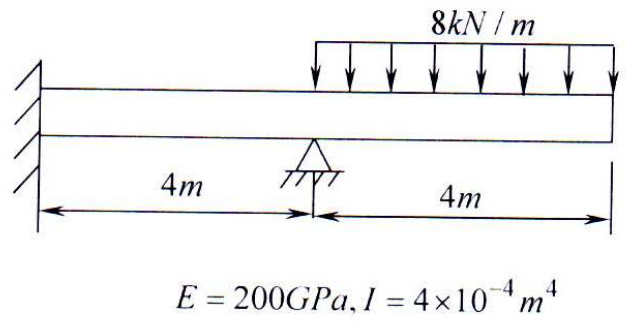


Fig. Q 7(b)

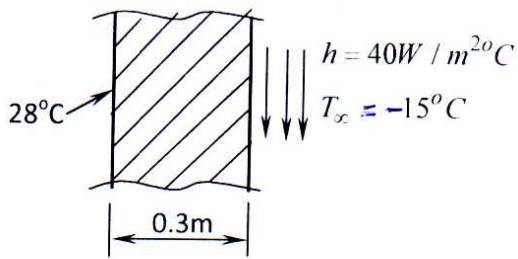


Fig. Q 9(b)

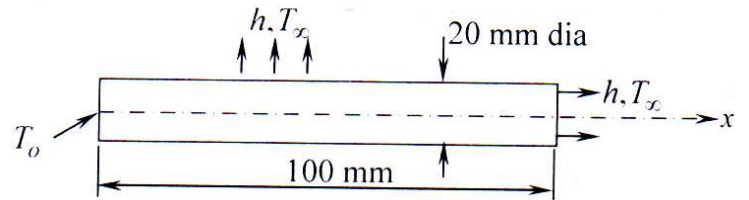


Fig. Q 10

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