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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Computer Science and Engineering

Semester End Examination; Dec - 2017/Jan - 2018

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. How many six digit integers (with no leading zero) are possible if,
i) No digits may be repeated ii) Digits may be repeated
iii) Answer part (i) and (ii) with extra condition that the digits are divisible by 5. 6

b. How many ways can a gambler draw five cards from a standard deck and get
i) Five cards of the same suit ii) Four aces
iii) Three of same kind and a pair of same kind iv) Three aces and two jacks. 8

c. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where
i) $x_i \geq 0, \quad 1 \leq i \leq 4$ ii) $x_i > 0, \quad 1 \leq i \leq 4$ iii) $x_1, x_2 \geq 5, \quad x_3, x_4 \geq 7$. 6

2 a. For U = {1, 2, 3,…….9, 10} Let A = {1, 2, 3, 4, 5}, B = {1, 2, 4, 8}, C = {1, 2, 3, 5, 7} and D = {2, 4, 6, 8}. Determine each of the following :
i) $(A \cup B) \cap C$ ii) $\overline{C} \cup \overline{D}$ iii) $(A \cup B) - C$ iv) $B - (C - D)$. 8

b. In a class of 50 college freshmen, 30 are studying C++, 25 are studying Java and 10 are studying both languages. How many freshmen are studying either computer language? 6

c. Consider drawing five cards from a standard deck of 52 cards. What is the probability of getting?
i) Three aces and two jacks ii) Three aces and a different pair. 6

UNIT - II

3 a. Construct the truth table to verify the following compound statement is a Tautology:
$\left[(p \to q) \wedge (q \to r)\right] \to (p \to r)$. 6

b. Provide the steps and reasons to establish the following equivalence:
$(p \to q) \wedge \left[\neg q \wedge (r \vee \neg q)\right] \Leftrightarrow \neg(q \vee p)$ 6

c. Establish the validity of the following argument by expression in symbolic form "If the band could not play rock music or the refreshments were not delivered on time then the new year's party would have been canceled and Alicia would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. Therefore the band could play rock music". 8

4 a. For the universe of all integers let $p(x), q(x), r(x), s(x)$ *and* $t(x)$ be the following:

$p(x): x > 0$ $q(x): x$ is even $r(x): x$ is a perfect square

$s(x): x$ is divisible by 4 $t(x): x$ is divisible by 5.

Write the following in symbolic form. Verify the statement is true or false; 10

i) At least one integer is even ii) If x is even then x is not divisible by 5

iii)No even integer is divisible by 5 iv) There exist an even integer divisible by 5

v) If x is even and x is a perfect square. Then x is divisible by 4.

b. Provide the steps and reasons to establish the validity of the argument: 6

$\neg r(c)$
$\forall x\left[p(x) \to q(x)\right]$
$\forall x\left[q(x) \to r(x)\right]$
$\therefore \neg p(c)$

c. For all integers *k* and *l*, if *k* and *l* are both odd, then their product *kl* is also odd 4

UNIT - III

5 a. For all $n \in z^{+}$ prove that, $\sum_{i=1}^{n} i = 1+2+3+.......+n = \frac{n(n+1)}{2}$. 6

b. Express the given sequence recursively and explicitly : 8

i) 2, 6, 12, 20 ii) 2, 10, 30, 68, …..

c. Prove that for any sets A, B, C $\subseteq$ U: 6

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ ii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

6 a. Let A = {1, 2, 3, 4} and B = $\{x, y, z\}$,

i) List five functions from A to B

ii) How many functions $f: A \to B$ are there?

iii) How many functions $f: A \to B$ are one to one? 6

iv) How many functions $g: B \to A$ are there?

v) How many functions $f: A \to B$ satisfy $f(1) = x$?

vi) How many functions $f: A \to B$ satisfy $f(1) = f(2) = x$?

b. Prove that for each $n \in z^{+}$, a sequence of $n^2 + 1$distinct real numbers contains a decreasing or increasing subsequence of length $n+1$ 8

c. Let f: R→R defined by 6

$$f(x) = \begin{cases} 3x-5 & x > 0 \\ -3x+1 & x \le 0 \end{cases}$$

Contd…3

i) Determine: $f(1)$, $f(-1)$ ii) Find $f^{-1}(0), f^{-1}(-3)$ iii) Find $f^{-1}([-5,5])$.

UNIT - IV

7 a. Prove that the number of symmetric relation on set A is, $2^{\frac{1}{2}(n^2+n)}$. 5

b. If A = {1, 2, 3, 4, 5} give an example of relation R on A that is
i) Reflexive and symmetric but not transitive ii) Reflexive and transitive but not symmetric iii) Symmetric and transitive but not reflexive. 6

c. State Topological sorting Algorithm. 4

d. Let A = $\{1, 2, 3, 6, 9, 18\}$ and define R on A if x/y verify it is poset? Draw Hasse diagram. 5

8 a. Define R on A = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12} by $(x, y) \in R$ if $x - y$ is a multiple of 5
i) Prove that R is an equivalence relation on A 8
ii) Determine the equivalence classes and partition of A induced by R.

b. Let A = {1, 2, 3, 4, 5, 6, 7} How many symmetric relation on A contain exactly,
i) Three ordered pair's ii) Five ordered pairs. 8

c. Let A = {1, 2, 3, 5, 30}, Prove that $(A, /)$ is a lattice by constructing a meet joint table. 4

UNIT - V

9 a. In the group S_6 let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 6 & 2 & 5 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 2 & 6 & 1 & 3 & 5 \end{pmatrix}$$ 10

Determine; $\alpha\beta$, α^3, $(\alpha\beta)^{-1}$, $\beta^{-1}\alpha^{-1}$.

b. Prove that $(M, \bullet)$ is an abelian group where $M = \{A, A^2, A^3, A^4\}$ with $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $\bullet$ is Matrix multiplication and also prove that $(M, \bullet)$ is isomorphic to the abelian group $(G, *)$ where $G = \{1, -1, i, -i\}$ and * is ordinary multiplication. 10

10 a. i) For $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$ find the subgroup $K = \langle \beta \rangle$ 8
ii) Determine left cosets of K in G = S_4.

b. A binary symmetric channel has probability P = 0.01 of incorrect transmission if C = 1011011101. What is the probability that,
i) We receive r = 0111111011 ii) We receive r = 1111011100 12
iii) A single error occurs iv) A double error occurs.

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