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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Information Science and Engineering
Semester End Examination; Dec - 2017/Jan - 2018
Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Define the following terms with an example for each. 6

i) Sum and Product Rule of Counting theory ii) Permutations and Combinations.

b. In how many ways can we distributed 10 identical marbles among six children such that 7

i) No restriction ii) Each child is given a marble

iii) First child gets exactly 5 marbles.

c. State Binomial theorem and determine the coefficient of $x^4\ y^4\ z^{20}$ in the expression $(4x - \frac{y}{2} + 3z + 5)^{30}$ 7

2 a. Using membership table method verify for any three sets A, B, C 6

$A-(B \cup C) = A-(B-C) = (A-B)-C.$

b. Find the number of integers between 1 to 1000 which are: 7

i) Not divisible by 2 or 3 or 5 ii) Exactly divisible by one of 2,3,5.

c. Two tickets are selected at random out of 50 tickets numbered 1, 2, 3……….50. What is the probability that the sum of the numbers of the tickets chosen is even? 7

UNIT - II

3 a. Prove law of syllogism and law of detachment to be tautological statements. 6

b. Prove logical equivalence without using truth table. 6

$i)\ p \wedge (p \vee (p \wedge q)) \equiv p$ $ii) [(p \to q) \to r] \equiv (\neg p \vee q) \to r)$

c. Text the validity of the argument. If I study then I will not fail in Maths. If I don’t play basketball, then I will study. But I failed in Maths. Therefore I must have played Basketball”. 8

4 a. Negate the quantified statements and represent each with a unique universe. 6

i) All birds have wings ii) All real numbers are complex numbers

iii) Some integers are divisible by 5 iv) Some triangles are equilateral.

b. Let, p(x): *x* is even q(x) : *x* is prime. 6

Find the truth value of;

$i) \forall x\ p(x)$ $ii) \exists q(x)$ $iii) \forall x\ p(x) \wedge q(x)$

$iv) \exists_x p(x) \to \neg q(x)$ $v) \forall x\ p(x) \vee q(x)$ $vi) \exists_x p(x) \wedge q(x)$

c. Express the argument in symbolic form and check its validity "No Engineering student is bad in studies. Anil is not bad in studies. Therefore Anil is an Engineering student". 8

UNIT - III

5 a. Prove by Mathematical Induction Principle that. 6

$n! \geq 2^{n-1} \forall n \geq 1.$

b. "If n is an odd integer then *n+11* is an even integer". Give a, 6

i) Direct proof ii) Indirect Proof iii) Proof by contradiction.

c. Let L_n denote n^{th} Lucus number and F_n denote n^{th} Fibonacci number then,

Prove that $L_n = F_{n-1} + F_{n+1} \quad \forall n \in Z^+$ 8

$\left[\text{Hint } L_0 = 2, L_1 = 1 \text{ and } L_n = L_{n-1} + L_{n-2} \forall\, n \geq 2\right].$

6 a. *Let* $|A| = m$ *and* $|B| = n$ *find*

i) How many functions are there from A and B 6

ii) Among them how many are one-one

iii) Among them how many are onto.

b. Find the number of ways of distributing 7 distinct objects into four labelled containers such that no container is left empty? 6

c. Consider the functions *f* and *g* defined by

$f(x) = x^3 \quad and \quad g(x) = 3x + 5y \ \forall x \in \mathbb{R}$

$h(x) = \begin{cases} 2x+1 & x \geq 1 \\ -2x+5 & x < 1 \end{cases}$ 8

Find $f \circ g,\ g \circ f,\ f^2,\ g^2,\ f \circ g \circ h,\ h \circ g \circ f,\ h^2,\ g \circ h.$

UNIT - IV

7 a. Define an equation relation. Prove that if R is a relation defined as: (a, b) $\in$ R, iff $a^2 = b^2$ on A = {-2, -1, 0, 1} is an equivalence relation and also find the partition induced by R on A. 7

b. Prove the following statements:

i) If R and S are equivalence relations on A Then $R \cap S$ is also equivalence 7

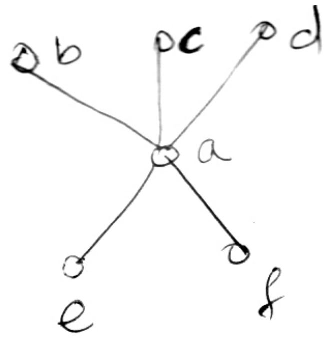
ii) If R and S are equivalence relatrions $R \cup S$ need not be an equivalence relation on A

c. Let A = {1, 2, 3, 4} and R = {(1, 2) (1, 3) (2, 4) (3, 2) (3, 3) (3, 4)}. Find R^2 and R^3 and write down their matrix and diagraphs. 6

8 a. Draw the Hasse's diagram representing the POSET (A, R) 7

Where A = {1, 2, 3, 6, 20, 36, 50} and "R" is "exactly divides" and Prove that R is a partially ordered relation.

b. Define the following terms and find the same for the graph given representing a POSET (A, R) 7



i) Least ii) Greatest

iii) LUB {a, e} iv) GLB {a, f}.

c. Write the steps involved in topological sorting algorithm. 6

UNIT - V

9 a. If o is an operation on $\mathbb{Z}$ defined by $x \circ y = x + y + 1$ Prove that ($\mathbb{Z}$,0) is an abelian group. 7

b. Prove that the group (Z_4, +) is cyclic. Find all its generators. 7

c. Define homomorphism, Isomorphism between two groups G_1 and G_2 with an example for each. 6

10 a. Explain encoding and decoding of a message with usual notations. 6

b. An encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$ 8

i) Find the associated parity check matrix H

ii) Use H to decode the received words 11101, 11011

c. Prove that $\forall$ x, y $\in$ $Z_2^m, wt(x+y) \leq wt(x) + wt(y)$. 6

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