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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester, B.E. - Semester End Examination; Dec - 2017/Jan - 2018

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs Max. Marks: 100

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find the n^{th} derivative of :

(i) $y=\log_{10}\left[(1-2x)^3(8x+1)^5\right]$ (ii) $y=e^{2x}\sin^3 x$. 6

b. Find the n^{th} derivative for the function $y=\dfrac{x^2-4x+1}{x^3+2x^2-x-2}$. 7

c. If $y=\left(\sin^{-1}x\right)^2$ show that, $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. 7

2 a. Find the angle of intersection of curves $\dfrac{2a}{r}=1+\cos\theta$ and $\dfrac{2b}{r}=1-\cos\theta$. 6

b. Find the Pedal equation of the curve $r^n=a^n\cos n\theta$. 7

c. Prove that the radius of curvature for the curve $r(1-\cos\theta)=a$ is $2\sqrt{\dfrac{2r^3}{a}}$. 7

UNIT - II

3 a. Apply Rolle's theorem and verify the same for the function:

$f(x)=e^x(\sin x-\cos x)\ in\left[\frac{\pi}{4},\frac{5\pi}{4}\right]$. 6

b. State Lagrange's mean value theorem and verify the same for $f(x)=x(x-1)(x-2)$ in $\left[0,\ \frac{1}{2}\right]$. 7

c. State Cauchy's mean value theorem and verify the Cauchy's mean value theorem for :

$\log_e x$ and $\dfrac{1}{x}$ in [1, e]. 7

4 a. Expand Taylor's series expansion of $\cos x$ about $x=\dfrac{\pi}{3}$ upto four non-zero terms. 6

b. Obtain the Maclaurin's series of $\log(1+x)$ and hence decuce that,

$\log\sqrt{\dfrac{(1+x)}{(1-x)}}=x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+.........$ 7

c. Find the values of a and b such that, $\lim\limits_{x\to 0}\left(\dfrac{x(1+a\cos x)-b\sin x}{x^3}\right)=1$. 7

UNIT - III

5 a. If $u=\log\left(x^3+y^3+z^3-3xyz\right)$ then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$. 6

b. *If* $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that using Euler's theorem : 7

(i) $xu_x+yu_y=\sin 2u$ (ii) $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}=\sin 4u-\sin 2u$.

c. If $H=f\left(x-y,\ y-z,\ z-x\right)$ show that $\frac{\partial H}{\partial x}+\frac{\partial H}{\partial y}+\frac{\partial H}{\partial z}=0$. 7

6 a. A particle moves along the curve $x=t^3+1$, $y=t^2$, $z=2t+3$ where 't' is the time. Compute the time components of its velocity and acceleration at t = 1 in the direction $\hat{i}+\hat{j}+3\hat{k}$. 6

b. Find the directional derivative of $\phi=x^2yz+4xz^2$ at (1, -2, -1) along $2\hat{i}-\hat{j}-2\hat{k}$. 7

c. *If* $\vec{F}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$ show that curl $\vec{F}.\vec{F}=0$. 7

UNIT - IV

7 a. Obtain the reduction formula for: $\int \sin^n x dx$ and $\int_0^{\pi/2} \sin^n x dx$. Where n is positive integer. 6

b. Evaluate: $\int_0^{\pi/6} \cos^4 3x.\sin^3 6x\, dx$ using reduction formula. 7

c. Trace the curve $y^2(2a-x)=x^3,\quad a>0$. 7

8 a. Find the length of an arch of the cycloid $x=a(t-\sin t)$, $y=a(1-\cos t)$. 6

b. Find the area of enclosed by the Astroid $x^{2/3}+y^{2/3}=a^{2/3}$. 7

c. By using the rule of differentiation under the integral sign, evaluate $\int_0^1 \frac{x^\alpha-1}{\log x}dx,\ \alpha\geq 0$. 7

UNIT - V

9 a. Solve : $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+\left[x+\log x-x\sin y\right]dy=0$. 6

b. Solve: $(x-4y-9)dx+(4x+y-2)dy=0$. 7

c. Solve: $\left(1+x^2\right)dy=\left(\tan^{-1}x-y\right)dx$. 7

10 a. Solve: $\left(x^2+y^2+x\right)dx+xydy=0$. 6

b. Find the orthogonal trajectory of the family of curve $y^2=4a(x+a)$. 7

c. A body originally at 100ºC cools down to 70ºC in 15 minutes, the temperature of the air being 30ºC. What will be the temperature of the body after 40 minutes from the original? 7

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