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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; June - 2017

Engineering Mathematics - IV

(Common to EE, EC, CS & E, IS & E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1\. a. Find a root of the equation $x^3 - 5x + 1 = 0$, using the bisection method in five stages. 6

b. Find the root of the equation $xe^x - \cos x = 0$, by the method of false position. Carryout 3 iterations. 7

c. Using Newton-Raphson method find, an approximate root of the equation $x \log_{10} x = 1.2$ correct to five decimal places that is near 2.5. 7

2 a. Using Modified Euler's method find $y(20.2)$, given that :

$$\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right) \text{ with } y(20) = 5 \text{ taking } h = 0.2.$$ 6

b. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$. 7

c. Find the value of y at $x = 4.4$, by applying Adams-Bash forth method given that :

$$5x\frac{dy}{dx} + y^2 - 2 = 0,\ y(4) = 1,\ y(4.1) = 1.0049,\ y(4.2) = 1.0097,\ y(4.3) = 1.0142.$$ 7

UNIT - II

3 a. Define Rank and Nullity of linear transformation and give examples. 6

b. Show that the three vectors (1, -3, 2), (2, 4, 1) and (5, -1, 0) are linearly independent in $V_3(R)$. 7

c. T is the linear transformation $V_3(R) \to V_3(R)$ defined by, $(x, y, z)T = (x - y,\ 2x + z,\ 2y)$. Find the matrix representation of T. 7

4 a. Employ Gauss-Seidel iteration method to solve,

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

Carryout 4 iterations. 6

b. Solve the following system of equations by relaxation method,

$$12x + y + z = 31$$
$$2x + 8y - z = 24$$
$$3x + 4y + 10z = 58.$$ 7

Contd…2

c. Find the numerically largest Eigen value and the corresponding Eigen vector of the matrix $A=\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$, by taking $[1, 0.8, -0.8]^T$ as the initial Eigen vector up to 5 approximations. 7

UNIT - III

5 a. Show that $f(z)=z^n$ is analytic and hence find its derivative (where n is positive integer). 6

b. Find the analytic function $f(z)$ as a function z, given that the sum of its real and imaginary parts is $x^3-y^3+3xy(x-y)$. 7

c. Find the Bilinear transformation which maps $z=\infty, i, 0$ into $w=-1,-i, 1$. Also find the invariant points of the transformation. 7

6 a. State Cauchy's integral formula and hence evaluate $\int_C \frac{e^{2z}dz}{(z+1)(z-2)}$ where $c:|z|=3$. 6

b. Expand $f(z)=\frac{z+1}{(z+2)(z+3)}$ in a Laurent's series valid for, i) $|z| > 3$ ii) $2 < |z| < 3$. 7

c. Evaluate $\int_c \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}dz$ where C is the circle $|z| = 3$. Using Cauchy's residue theorem. 7

UNIT - IV

7 a. The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Calculate the skewness and kurtosis. 6

b. An experiment on lifetime t of cutting tool at different cutting speeds v (units) is given below : 7

Speed *(v)*	350	400	500	600
Life *(t)*	61	26	7	2.6

Fit a relation of the form $v = a.t^b$.

c. Find the correlation coefficient and the equation of the lines of regression for the following values of x and y : 7

x:	1	2	3	4	5
y:	2	5	3	8	7

8 a. A random variable X has the following probability function for various values of x. 6

x	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find K ii) Evaluate $P(x<6), P(3<x\le 6)$.

Contd…3

b. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of, 7

i) No error during a micro second

ii) One error per micro second

iii) At least one error per micro second.

c. In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and S.D. of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for : 7

i) More than 2100 hrs ii) Less than 1950 hrs

Given data: $\phi(1.67) = 0.4525$, $\phi(0.83) = 0.2967$.

UNIT - V

9 a. Suppose X and Y are independent random variables with the following respective distribution, find the joint distribution of X and Y. Also verify that COV(X, Y) = 0 6

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

b. Find the unique fixed probability vector of the regular stochastic matrix. 7

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

c. The joint density function of two continuous random variables X and Y is given by, 7

$$f(x) = \begin{cases} kxy, & 0 \le x \le 4 \quad 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find: i) The value of K ii) E(XY) iii) E(2X + 3Y).

10 a. Obtain the series solution of the equation $\frac{d^2y}{dx^2} + xy = 0$. 6

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}.\sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. 7

c. If $x^3 + 2x^2 - x + 1 = ap_0(x) + bp_1(x) + cp_2(x) + dp_3(x)$ find the values of a, b, c, d. 7

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