



P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Sixth Semester, B.E. – Mechanical Engineering

Semester End Examination; June - 2017

Finite Element Methods

Time: 3 hrs

Max. Marks: 100

Note: (i) Answer **FIVE** full questions, selecting **ONE** full question from each **unit**.
 (ii) Answer missing data, if any may be suitably assumed.

UNIT - I

- 1 a. Explain the following with suitable example:
 - (i) Boundary value problem and initial value problem 8
 - (ii) Essential boundary conditions and non-essential conditions.
- b. Define body force and traction force and give any two examples for each. 4
- c. With suitable example, explain the concept of plane stress and plane strain problems. Write stress-strain relations for each problem. 8
- 2 a. Explain how node numbering scheme in the discretization process affects bandwidth of stiffness matrix. 4
- b. Solve the following system of simultaneous equations using general algorithms of gauss elimination method. 10

$$x_1 + x_2 + x_3 = 9$$

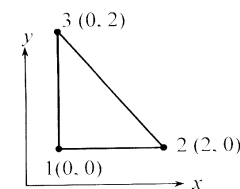
$$x_1 - 2x_2 + 3x_3 = 8$$

$$2x_1 + x_2 - x_3 = 3$$
- c. Evaluate the integral using Gauss quadrature so that the result is exact. 6

$$I = \int_{-1}^1 (7 - 3x + x^2) dx$$

UNIT - II

- 3 a. Briefly explain the convergence criteria of a displacement function. 6
- b. Derive shape functions for a 3-noded triangular element in terms of natural co-ordinate systems. 6
- c. Derive Jacobean matrix for a CST element and use this expression to determine Jacobean for the triangular plate shown FigQ.3(c).



All coordinates are in mm

Fig. Q3(c)

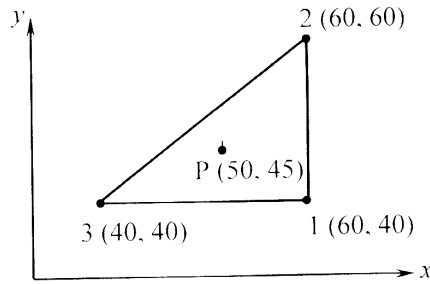
4 a. Write a note on Lagrangian interpolation function and derive functions for a 4-noded quadrilateral element using it. 12

b. The nodal displacements of a triangular element shown in Fig.Q4(b) are given by,

$$u_1 = 0.0 \text{ mm}; u_2 = 0.03 \text{ mm}; u_3 = 0.0 \text{ mm}$$

$$v_1 = 0.0625 \text{ mm}; v_2 = 0.0 \text{ mm}, v_3 = 0.0625 \text{ mm}$$

Determine the displacement at a point P whose x and y coordinate are (50, 45) mm.



All coordinates are in mm

Fig. Q 4(b)

8

UNIT - III

5 a. A 2-noded bar element is subjected to a body force (force/unit volume) and surface forces (force/unit length), derive expressions for element load vectors due to the above forces. 10

b. Derive strain-displacement matrix B and strain matrix for a linear triangular element and show that they are constant. 10

6. A stepped bar member is loaded as shown in Fig.Q.6. Determine the nodal displacement, stresses in each bar and support reactions. Use Penalty method to handle the boundary conditions. Take $E = 200 \text{ GPa}$.

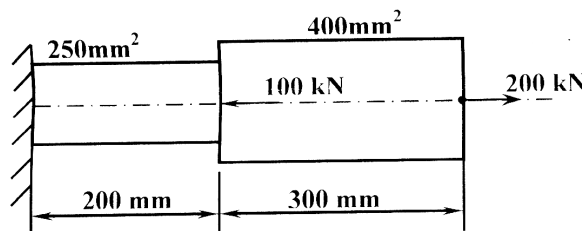


Fig. Q 6 ✓

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UNIT - IV

7 a. What is transformation matrix? Mention how it is useful in the analysis of truss problems. 4

b. For the truss structure shown in Fig.Q7 (b), determine the nodal displacements, stress in horizontal member and reactions at top support. Take $E = 200 \text{ GPa}$ and $A = 200 \text{ mm}^2$

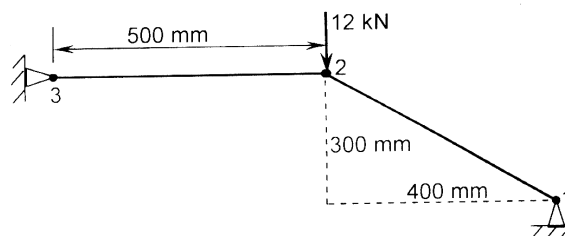
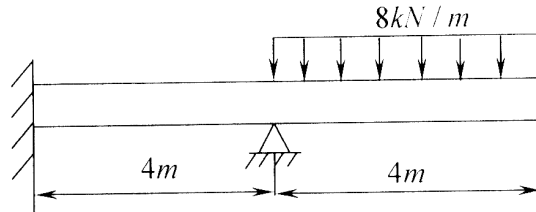


Fig. Q 7(b) ✓

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- 8 a. Write the expressions of Hermite's shape functions of a 2-noded beam element and draw their variation along the element. Why they are called Hermite's shape functions?
- b. For the beam shown in Fig. Q.8(b). Determine the nodal deflections and slopes. Take $E = 70 \text{ GPa}$, $I = 3 \times 10^{-4} \text{ m}^4$



$$E = 200 \text{ GPa}, I = 4 \times 10^{-4} \text{ m}^4$$

Fig. Q 8(b)

UNIT - V

- 9 a. Using Galerkin's approach, derive the element conditions matrix for 1D element used for steady state heat transfer problems.
- b. Consider a brick wall Fig. Q.9(b) of thickness, $L = 0.3 \text{ m}$, $K = 0.7 \text{ W/m}^\circ\text{C}$. The inner surface is at 28°C and the outer surface is exposed to cold air at -15°C . The heat transfer coefficient associated with the outside surface is $h = 40 \text{ W/m}^2\text{K}$. Determine the steady-state temperature distribution within the wall and also the heat flux through the wall. Use a two-element model.

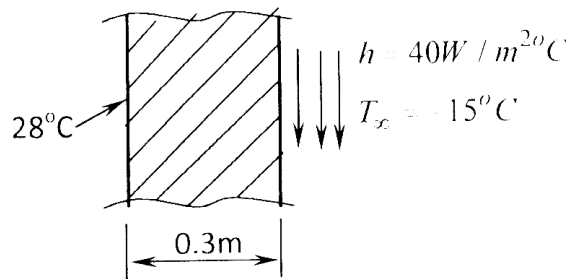


Fig. Q 9(b)

10. A metallic fin, shown in Fig. Q.10. with thermal conductivity $K = 360 \text{ W/m}^\circ\text{C}$, 0.1 cm thick and 10 cm long extends from a wall whose temperature is 235°C . Determine the temperature distribution and amount of heat transferred from the fin to air at 20°C with $h = 9 \text{ W/m}^2 \text{ K}$. Take the width of fin to be 1 m and use three-element model.

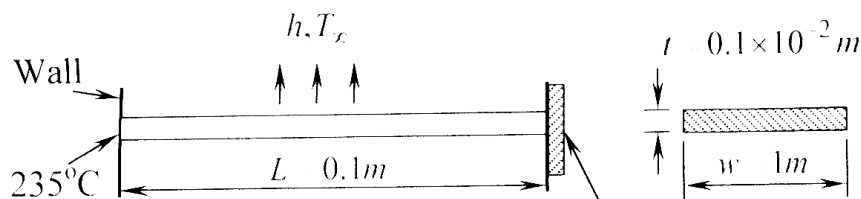


Fig. Q10

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