



P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Sixth Semester, B.E. - Mechanical Engineering

Semester End Examination; June - 2017

Theory of Elasticity

Time: 3 hrs

Max. Marks: 100

Note: i) Answer **FIVE** full questions, selecting **ONE** full question from each unit.

ii) Missing data may suitably assume.

UNIT - I

- 1 a. Determine the principal stresses and principal direction for their major axes for the state of stress by the following stress matrix

$$\sigma_{ij} = \begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix} \quad 12$$

- b. Derive an equation of equilibrium for 3D state of stress. 8

- 2 a. Stress components are given by $\sigma_x = \sigma_y = \sigma_z = 100$, $\tau_{xy} = 0$, $\tau_{zy} = 400x$, $\tau_{zx} = 200y$ 12
Determine normal and shear stress at point (1,2,3) on a plane $x + y + z = 6$.

- b. Derive an equation to find the stress components on an arbitrary plane. 8

UNIT - II

- 3 a. The strain tensor at a point in a body is given by

$$\epsilon_{ij} = \begin{bmatrix} 0.0001 & 0.0002 & 0.0005 \\ 0.0002 & 0.0003 & 0.0004 \\ 0.0005 & 0.0004 & 0.0005 \end{bmatrix} \quad 12$$

Determine;

- (i) Octahedral normal and shearing strain (ii) Deviatoric and spherical strain tensor.

- b. Derive strain-displacement relation. 8

- 4 a. The displacement at point given by $u = (x^2 + 3) \times 10^{-3}$, $v = 3y^2z \times 10^{-3}$, $w = (x + 3z) \times 10^{-3}$ 6
Determine the strain components at point (1,2,3)

- b. For a given strain field,

$$\begin{aligned} \epsilon_x &= 5 + x^2 + y^2 + x^4 + y^4; & \epsilon_y &= 6 + 3x^2 + 3y^2 + x^4 + y^4; \\ \gamma_{yz} &= 10 + 4xy(x^2 + y^2 + 2); & \epsilon_z &= \gamma_{yz} = \gamma_{zx} = 0 \end{aligned} \quad 10$$

Determine if this system of strain is possible and find displacement components in terms of x, y assuming their displacement and rotation about origin is zero.

- c. List the six compatibility equations. 4

UNIT - III

- 5 a. A cubical element is subjected to the following state of stress:
 $\sigma_x = 100\text{MPa}$, $\sigma_y = -20\text{MPa}$, $\sigma_z = -400\text{MPa}$, $\tau_{xy} = \tau_{zy} = \tau_{zx} = 0$ 10
Assuming material to be homogeneous and isotropic, determine the principal shear strains and the octahedral shear strain, if $E = 2 \times 10^5 \text{ MPa}$ and $\mu = 0.25$.
- b. The displacement field is given by $u_x = 0.06x + 0.05y - 0.01z$, $u_y = 0.01y - 0.03z$, $u_z = -0.02x + 0.01z$ at point (1, 2, 3). Determine stress components assuming $E = 196 \text{ GPa}$ and $G = 78.48 \text{ GPa}$. 10
6. Discuss the following : 20
- i) Principle of superposition
 - ii) Hooke's law
 - iii) Uniqueness theorem
 - iv) Saint-Venant's principle.

UNIT - IV

- 7 a. Define the stress function. Derive the equations for stress function for the plane strain case. 10
- b. Obtain expressions for the torsion in a bar of circular cross section. 10
- 8 a. Given the stress function $\phi = \left(\frac{H}{\pi}\right) z \tan^{-1}\left(\frac{x}{z}\right)$. Determine whether stress function ϕ is admissible. If so determine the stresses. 10
- b. Obtain expression for the torsion in a bar of elliptical cross section. 10

UNIT - V

9. Derive the equation for thick walled cylinder subjected to internal and external pressures. 20
- 10 a. Determine the stress in thin circular disk subjected temperature symmetrical about centre. 10
- b. Show that the resultant circumferential force across any radial section of a hollow disk subjected to thermal loading is zero. 10

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