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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Computer Science and Engineering

Semester End Examination; June - 2017

Graph Theory and Combinatories

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. In an undirected graph G = (V, E) with |V| = v and |E| = e and no loops, show that $2e \leq v^2 - v$. What is the corresponding result when G is directed? 7

b. Let G = (V, E) be a loop-free connected undirected graph, and let {a, b} be an edge of G. Prove that {a, b} is part of a cycle if and only if its removal (the vertices a and b are left) does not disconnect G. 6

c. i) If G_1, G_2 are (loop-free) undirected graphs, prove that G_1, G_2, are isomorphic if and only if $\overline{G_1}, \overline{G_2}$ are isomorphic.

ii) If G = (V, E) is an undirected graph or multigraph prove that $\sum_{v \in V} \deg(v) = 2|E|$. 7

2 a. Let $n \in Z^+$, with $n \geq 9$. Prove that if the edges of K_n can be partitioned into sub graphs isomorphic to cycles of length 4 (where any two such cycles share no common edge) then n = 8K+1 for some $K \in Z^+$. 5

b. Explain why it is not possible to draw a loop-free connected undirected graph with eight vertices, where the degrees of the vertices are 1, 1, 1, 2, 3, 4, 5 and 7. 7

c. Write a short note on Konigsberg's bridge problem. 8

UNIT - II

3 a. i) Prove that K_5 is non planar.

ii) Prove that $K_{3,3}$ is non planar. 8

b. How many vertices and how many edges are there in the complete bipartite graphs $K_{4,7}$, $K_{7,11}$ and $K_{m,n}$, where m, n $\in Z^+$? 5

c. Prove that every loop-free connected planar graph has a vertex v with $\deg(v) < 6$. 7

4 a. Prove that if G=(V, E) is a connected graph and e$\in E$ then, $P(G_e, \lambda) = P(G, \lambda) + P(G'_e, \lambda)$ 8

b. Explain Welsh and Powell graph coloring algorithm. 5

c. Determine P(G, λ) for G = $K_{1,3}$. 7

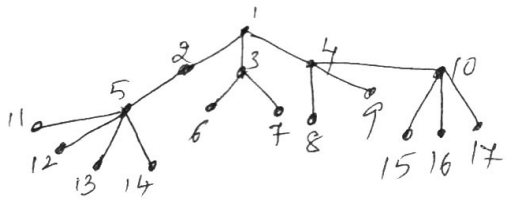
UNIT - III

5 a. If a, b are distinct vertices in a tree T = (V, E) then prove that there is a unique path that connects these vertices. 5

b. Show that all trees are planar. 5

Contd…2

c. Write pre-order, post-order, traversal for the following tree. 5



d. Explain the steps involved in merge sort algorithm. 5

6 a. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. 5

b. Explain the steps involved in Dijkstra's algorithm. 5

c. Explain Prim's algorithm. 5

d. Explain Kruskal's algorithm. 5

UNIT - IV

7 a. Buick automobiles come in four models, 12 colours, three engine sizes and two transmission types.

(i) How many distinct Buicks can be manufactured?

(ii) If one of the available colours is blue how many different blue Buicks can be manufactured? 5

b. i) How many permutations for eight letters a, c, f, g, i, t, w, x.

ii) Consider the permutations in part <i>. How many start with letter t? How many starts with letter t and end with letter c? 5

c. Find the number of arrangements of the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's? 5

d. How many bytes contain :

(i) Exactly two 1's (ii) exactly four 1's (iii) exactly six 1's (iv) at least six 1's. 5

8 a. How many integer solutions are there for the equation $C_1+C_2+C_3+C_4=25$, if $0 \le C_i$, for all $1 \le i \le 4$? 6

b. Determine the co-efficient of x^8 in $\dfrac{1}{(x-3)(x-2)^2}$. 7

c. Find a formula for $\sum_{k=1}^{n} k$ using generating function for the sequence 0,1,3,6,10,15, ... 7

UNIT - V

9 a. Solve the recurrence relation $a_n = 7a_{n-1}$, where $n \ge 1$ and $a_2 = 98$. 8

b. Solve the recurrence relation $F_{n+2} = F_{n+1}+F_n$ where $n \ge 0$ and $F_0 = 0$, $F_1 = 1$. 7

c. Solve recurrence relation $2a_{n+3} = a_{n+2}+2a_{n+1}-a_n$, $n \ge 0$, $a_0 = 0$, $a_1=1$, $a_2=2$. 5

10 a. Solve recurrence relation $a_n-3a_{n-1} = 5(7^n)$, where $n \ge 1$ and $a_0 = 2$. 7

b. Solve the relation $a_n-3a_{n-1} = n$, $n \ge 1$, $a_0 = 1$. 7

c. Find number of n digit quaternary (0, 1, 2, 3) sequences in which there is never a '3'anywhere to the right of a '0'. 6

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