U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)
Fourth Semester, B.E. - Information Science and Engineering
Semester End Examination; June - 2017
Graph Theory and Combinatories

Time: 3 hrs   *Max. Marks: 100*

*Note: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

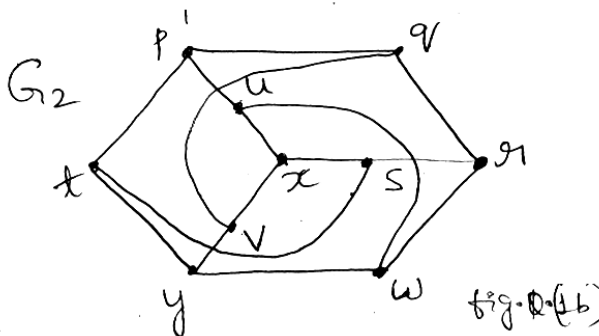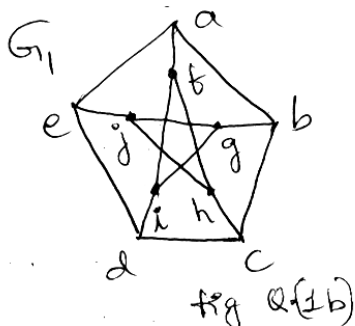1 a. Define :   6

   (i) Connected Graph   (ii) Spanning sub graph   (iii) Bipartite graph; Give one example.

  b. Define isomorphism. Show that the following graphs are isomorphic.   7

fig Q(1b)   fig.Q(1b)

  c. Write a short note on Konigsberg's bridge problem.   7

2 a. Let G be a simple graph of order n. If the size of G is 56 and the size of $\bar{G}$ is 80, what is *n*?   6

  b. Prove that in every graph, the number of vertices of odd degree is even.   7

  c. Define Hamiltonian cycle. How many edge disjoint Hamiltonian cycles exist in the
     complete graph with seven vertices? Also, draw the graph to show these Hamiltonian cycles
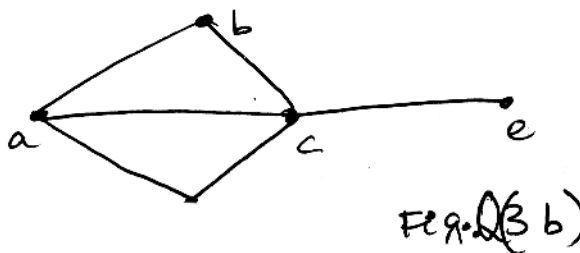   of K_7.   7

UNIT - II

3 a. Prove that Kuratowskis second graph is non planar.   7

  b. Find the geometric dual of the following graph Fig. Q 3(b).   6

Fig.Q(3 b)

  c. Prove that a connected planar graph with '*n*' vertices and '*m*' edges has exactly *m-n+2*
     regions.   7

4 a. Define chromatic number and determine chromatic number and chromatic polynomial for the give given below.

fig. Q(4a)

7

b. Define :

(i) cut set   (ii) edge connectivity   (iii) vertex connectivity;

Give one example for each.

6

c. Write the steps to detect planarity in a graph with an example.

7

UNIT - III

5 a. Apply Kruskal's algorithm to find a minimal spanning tree for the weighted graph below :
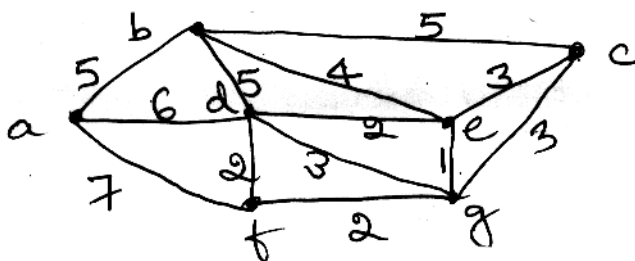

fig. Q(5a)

7

b. Obtain an optimal Prefix code for the message "FALL OF THE WALL".

7

c. Define (i) Tree   (ii) Rooted tree   (iii) Balanced Tree.

Give one example for each.

6

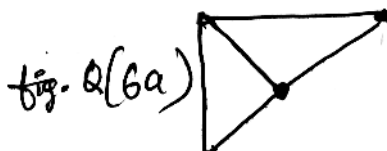6 a. Find all the spanning trees of the given graph.

Fig Q(6a)

6

b. Suppose that a tree 'T' has two vertices of degree 2, four vertices of degree 3 and three vertices of degree 4. Find the number of pendent vertices is T.

7

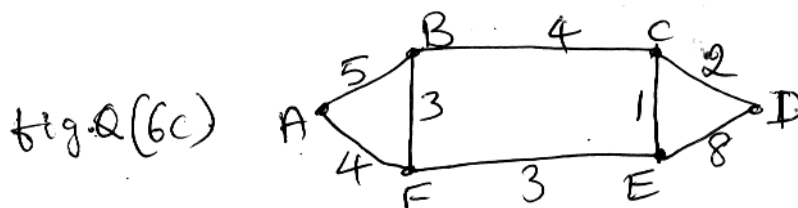c. Find the maximum flow between the two vertices A and D in the following graph.

fig.Q(6c)

7

UNIT - IV

7 a. Determine the number of Positive integers in such that $1 \le n \le 100$ and 'n' is not divisible by 2, 3 or 5.   7

 b. Define derangement. Find the number of derangement of 1, 2, 3, 4. List all the derangements.   6

 c. A girl has sarees of 5 different colors; blue, green, red, white and yellow. On Monday, she does not wear green; on Tuesdays blue or red on Wednesday blue or green, Thursday red or yellow; on Fridays, red. In how many color ways can she dress without repeating a color during a week?   7

8 a. Find a generating function for the following sequences :

 (i) 0^2, 1^2, 2^2, 3^2, 4^2, ………..   7

 (ii) 0, 2, 6, 12, 30, 30, 42…..

 b. Find the exponential generating function for the number of ways to arrange "n" letters selected from MISISSIPPI.   6

 c. Find the sequences generated by the following functions :

 (i) $(3+x)^3$ 7

 (ii) $2x^2(1-x)^{-1}$.

UNIT - V

9 a. Solve the recurrence relation $a_n = n\, a_{n-1}$ for $n \ge 1$, given that $a_0 = 1$.   6

 b. Find a recurrence relation and the initial condition for the sequence 2, 10, 50, 250……… Hence find the general term of the sequence   7

 c. The number of virus affected files in a system is 1000 (to start with and this increase 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day.   7

10 a. Solve the recurrence relation :

 $a_{n+1} - 6a_{n-1} + 9a_{n-2} = 0$ for $n \ge 2$ 6
 Given that $a_0 = 5, a_1 = 12$.

 b. Find the generating function for the recurrence relation: $a_{n+1} - a_n = n^2$ $n \ge 0$ *and* $a_0 = 1$ and hence solve it.   7

 c. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, … Hence find the general term of the sequence.   7

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