U.S.N

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; June - 2017

Engineering Mathematics - IV

(Common to AU, CV, ME & IP Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1. a. Show that $f(z)=\left(r+\frac{k^2}{r}\right)\cos\theta+i\left(r-\frac{k^2}{r}\right)\sin\theta,\ r\neq 0$ is a regular function of $z=re^{i\theta}$ and hence find $f'(z)$. 6

b. Show that $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic and find its harmonic conjugate. 7

c. Find the bilinear transformation which map the points z = 0, 1, ∞ into the points w = -5, -1, 3 respectively. What are the invariant points in this transformation? 7

2 a. Evaluate $\int_{(0,3)}^{(2,4)}\left(2y+x^2\right)dx+\left(3x-y\right)dy$ along the following paths : 6

i) The parabola $x = 2t$, $y = t^2 + 3$

ii) The straight line from (0, 3) to (2, 4).

b. Expand $f(z)=\dfrac{z+1}{(z+2)(z+3)}$ in a Laurent's series valid for : i) $|z|>3$ ii) $2<|z|<3$. 7

c. Evaluate $\int_c \dfrac{\left(z^2+5\right)dz}{(z-2)(z-3)}$ using Cauchy's residue theorem $C:|z|=4$. 7

UNIT - II

3 a. Find $\sqrt[3]{37}$ using Newton-Raphson method. Carry out three iterations. 6

b. Compute the real root of $x\log_{10}x-1.2=0$, by the method of false position. Carryout three iterations. 7

c. Find the real root of the equation $x^2+2x-2=0$, using fixed point iteration method. Accelerate the convergence by Aitken's Δ^2-method. 7

4 a. Using Euler predictor and Corrector formula to solve, 6

$\dfrac{dy}{dx}=x+y$ at $x=0.2,$ given that $y(0)=1$.

b. Use fourth order Runge-Kutta method to compute $y(1.1)$, given that $\dfrac{dy}{dx}=xy^{1/3}$, $y(1)=1$. 7

c. If $\frac{dy}{dx} = 2e^x - y,\ y(0) = 2,\ \ y(0.1) = 2.010, y(0.2) = 2.040$ and $y(0.3) = 2.090,$ find $y(0.4)$ correct to four decimal places by using Milne's Predictor-Corrector method. 7

UNIT - III

5 a. Find the skewness and kurtosis, if the first four moments of a frequency distribution about $x = 4$ are respectively 1, 4, 10 and 45. 6

b. Fit a parabola of second degree $y=ax^2+bx+c$ for the data : 7

x	0	1	2	3	4
y	1	1.8	1.3	2.5	2.3

c. Find the coefficient of correlation and obtain the lines of regression for the data : 7

x	5	7	8	10	11	13	16
y	33	30	28	20	18	16	9

6. a. A random variable x has the following Probability function for various values of x, 6

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find k

ii) $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$.

b. The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers find approximately the number of drivers with, 7

i) No accident in a year ii) More than 3 accidents in a year.

c. The marks of 1000 students in an examination fallows a Normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be : 7

i) Less than 65 ii) More than 75 iii) Between 65 and 75. (Given ϕ (1) = 0.3413).

UNIT - IV

7 a. Suppose X and Y are independent random variables with the following respective distribution, find the joint probability distribution of X and Y. Also verify that COV (X, Y) = 0. 6

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

b. If the Joint probability function of the random variables X and Y is given by, 7

$$f(x, y) = \begin{cases} cxy, & 0 \leq x \leq 2 \quad 0 \leq y \leq x \\ 0, & otherwise \end{cases}$$

Contd…3

Determine c and hence find $P\left(\frac{1}{2} < X < 1\right)$.

c. Find the unique fixed probability vector of the regular stochastic matrix,

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$ 7

8 a. Solve the system of equation by Gauss-Seidel method :

$8x+3y+2z=13, 2x+y+6z=9, x+5y+z=7$ starting from $[0,0,0]^T$ carry out four iterations. 6

b. Solve: $10x+2y+z=9,\quad x+10y-z=-22,\quad -2x+3y+10z=22$, using relaxation method. Taking $\Delta x=0,\quad \Delta y=0,\quad \Delta z=0.$ 7

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix,

$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by power method taking the initial Eigen vector as $[1,1,1]^T$. 7

UNIT - V

9 a. Derive Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0.$ 6

b. Show that an extremal of $\int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is expressible in the form $y = ae^{bx}$. 7

c. Find the Geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x\left(1+y^{1^2}\right)}dx$ 7

10 a. Obtain the series solution of the equation $\frac{d^2y}{dx^2} + x^2 y = 0$ 6

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ 7

c. If $x^3+2x^2-x+1 = aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$ Find the values of a, b, c, d. 7

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