***U.S.N*** | | | | | | | | | | |

# P.E.S. College of Engineering, Mandya - 571 401

***(An Autonomous Institution affiliated to VTU, Belgaum)***

## Third Semester, B.E. - Semester End Examination; Dec - 2016/Jan - 2017

## Engineering Mathematics - III

## (Common to all Branches)

*Time: 3 hrs* *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

**UNIT - I**

1 a. The Area (A) of a circle corresponding to diameter (D) is given below 7

| D: | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|
| A: | 5026 | 5674 | 6362 | 7088 | 7854 |

Find the value of A when D =105, using appropriate formula.

b. Use Lagrange's interpolation formula to fit a polynomial to the data. 7

| x | 0 | 1 | 3 | 4 |
|---|---|---|---|---|
| Y | -12 | 0 | 6 | 12 |

c. Given $u_{20} = 24.37, u_{22} = 49.28, u_{29} = 162.86$ *and* $u_{32} = 240.50$ find $u_{28}$ by using Newton's divided difference formula. 6

2 a. Find $f'(1)$ and $f''(3)$ given that 7

| x: | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| Y = *f*(x) | 7 | 13 | 43 | 145 | 367 |

b. Evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's $\left(1/3\right)^{rd}$ rule, taking seven equidistant ordinates. 7

c. Use Weddle's rule to evaluate $\int_4^{5.2} \log x dx$ by dividing $[4, 5.2]$ into six equal Parts. 6

**UNIT - II**

3 a. Obtain the Fourier series to represent $e^{-x}$ from $x = -\pi$ to $x = \pi$ 7

b. Find the Fourier series of the function $x^2$ valid in $-\pi \le x \le \pi$ Hence deduce that 7

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots..$$

c. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. 6

4 a. Express $f(x) = \begin{cases} 2 - x & in \quad 0 \le x \le 4 \\ x - 6 & in \quad 4 \le x \le 8 \end{cases}$ as a Fourier series and hence deduce that 6

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots..$$

b. Given : 7

| x | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|---|---|---|---|---|---|---|---|
| y | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Find y as a Fourier series up to the second harmonies

c. Find half-range cosine series of $f(x) = (x-1)^2, 0 \le x \le 1$ 7

## UNIT - III

5 a. If $f(x)=\begin{cases}1-x^2 & |x|<1\\ 0 & |x|\geq 1\end{cases}$ Find the Fourier transform of $f(x)$ and hence find the value of $\int_0^{\infty}\frac{x\cos x-sinx}{x^3}dx$ 6

b. Find the Fourier Sine and cosine transform of $e^{-ax}$ $a>0$. 7

c. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ , $a>0$ 7

6 a. Obtain the Z-transform of $coshn\theta$ and $sinhn\theta$ 7

b. Find the inverse Z-transform of $\frac{3z^2+z}{(5z-1)(5z+2)}$ 7

c. Solve the difference equation : $y_{n+1}+\frac{1}{4}y_n=\left(\frac{1}{4}\right)^n$ with $y_0=0$ 6

## UNIT - IV

7 a. Form the partial differential equation by eliminating the arbitrary functions from $z=f\left(x^2-y\right)+g\left(x^2+y\right)$ 7

b. Solve : $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given that $u(0,y)=2e^{5y}$ 7

c. Solve : $x\left(y^2+z\right)p-y\left(x^2+z\right)q=z\left(x^2-y^2\right)$ 6

8 a. Find the various possible solutions of one-dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. 10

b. Solve one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ given that $u(0,\ t)=0$, $u(l,\ t)=0$ and $u(x,0)=k\left(lx-x^2\right)$. 10

## UNIT - V

9 a. Test for convergence

i) $\sum\frac{1}{\sqrt{n+1}+\sqrt{n}}$ ii) $\sum\frac{(n+1)^n}{n^{n+1}}x^n\,(x>0)$ 7

b. Find the nature of the series

$\frac{2}{3}x+\frac{2.3}{3.5}x^2+\frac{2.3.4}{3.5.7}x^3+........$ 7

c. Discuss the nature of the series $1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\frac{1}{6}-\frac{1}{7}-\frac{1}{8}+........$ 6

10a. Obtain the series solutions of $\frac{d^2y}{dx^2}+xy=0$ 7

b. With usual notations, prove that

$(i)$ $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ $(ii)$ $J_{-1/2}(x)=\sqrt{\frac{2}{\pi x}}\cos x$ 7

c. Express $x^4-3x^2+x$ in terms of Legendre's Polynomial. 6

* * *