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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

Third Semester, B.E. - Information Science and Engineering

Semester End Examination; Dec - 2016/Jan - 2017

Discrete Mathematical Structure

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. In how many ways can eight men and eight women be seated in a row if,
(i) Any person may sit next to any other
(ii) Men and women must occupy alternate seats
(iii) Generalize the result for '*n*' men and '*n*' women. 6

b. (i) How many permutations are there for the eight letters a, c, f, g, i, w, t, *x*?
(ii) Consider the permutations in part (i). How many start with letter't'? How many start with letter't' and end with the letter 'c'? 7

c. (i) In how many ways can we select five books from a library of 10 consisting of one C++, one network, One Database, One file structure, one maths and five Data mining (identical)?
(ii) In how many ways can we select '*n*' objects from a collection of size *2n* that consisting of '*n*' distinct and '*n*' identical objects. 7

2 a. Define power set, subset, and null set of A with an example. 6

b. (i) How many rows are needed to construct the membership table for $A \wedge (B \cup C) \cap (D \cup \overline{E} \cup \overline{F})$?
(ii) Use membership tables to determine whether or not $(A \cap B) \cup (\overline{B \cap C}) = A \cup \overline{B}$. 6

c. (i) Two integers are selected at random and without replacement from {1, 2, 3, ………. 99,100}. What is the probability the integers are consecutive?
(ii) If Tom tosses a fair coin six times. What is the probability he gets all heads, one head, two heads, even number of heads and atleat four heads? 8

UNIT - II

3 a. Define Tautology. Is $(p \vee q) \rightarrow (p \rightarrow (p \wedge q))$ a tautology? Justify the answer using truth table and without using truth table. 6

b. If *p*, *q* are primitive statements, prove that $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow (p \wedge q)$ is logically equivalence and write the dual of the logical equivalence. 6

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c. Write the following arguments in symbolic form, then use resolution (along with-the rules of inference and the laws of logic) to establish its validity.
Radha does not have her driver's license or her new car is out of gas. Radha has her driver's licence or she does not like to drive her new car. Radha's new car is not out of gas or she does not like to drive her new car. Therefore, Radha does not like to drive her new car. 8

4 a. Write the statements in the symbolic form with a specific universe for each,
(i) All employees have greater than 50% salary
(ii) Some students have enrolled for 'C++' course 6
(iii) Some integers are divisible by 5 and are even.

b. Prove that for every integer n, if n is odd, then n^2 is odd. 7

c. Prove that for all real numbers x and y, if $x + y \geq 100$, then $x \geq 50$, or $y \geq 50$. 7

UNIT - III

5 a. Show that $2^n > n^2$ whenever n is +ve integers greater than 4. 7

b. If n is + integer prove that, 7
$1.2+2.3+3.4+.......n(n+1)=\frac{n(n+1)(n+2)}{3}$, using mathematical Induction.

c. Determine which of the following functions are one-to-one and find its range,
(i) $f:Z\to Z,\ f(x)=2x$
(ii) $f:R\to R,\ f(x)=e^{x^2}$ 6
(iii) $f:Q\to Q,\ f(x)=2x$.

6 a. Define Stirling's number of second kind. Evaluate S(6, 4). 6

b. Let f and g be two functions from R to R defined by $f(x)=2x+1$ and $g(x)=x/3$, 7
Find; (i) f o g and g o f (ii) $(g \circ f)^{-1}$ and $f^{-1} \circ g^{-1}$.

c. Define Pigeon hole principle. Prove that if 101 integers are selected from the set S = { 1, 2, 3, …..200} then there are two integers such that one divide the other. 7

UNIT - IV

7 a. What are the properties of Relations? List and explain with neat sketch. 6

b. Let A = {1, 2, 3}, B = {w, x, y, z} and C = {4, 5, 6}. Define the relations $R_1 \subseteq A\times B$, $R_2 \subseteq B\times C$ and $R_3 \subseteq B\times C$ where $R_1=\{(1,w)(3,w)(2,x)(1,y)\}$,
$R_2=\{(w,5)(x,6)(y,4)(y,6)\}$ and $R_3=\{(w,4)(w,5)(y,5)\}$. Determine; 8
(i) $R_1\circ(R_2\cup R_3)$ and $(R_1\circ R_2)\cup(R_1\circ R_3)$
(ii) $R_1\circ(R_2\cap R_3)$ and $(R_1\circ R_2)\cap(R_1\circ R_3)$.

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c. Write the relation matrix and directed graph for the relations defined as, 6

(i) "$x+y\geq 2y$" for the set given $A=\{2,3,4,6,7\}$.

8 a. Define maximal element, minimal element, lower bound and upper bound. 6

b. Draw the Hase diagram for the poset $\{P(u),\subseteq\}$ where $u=\{1,2,3,4\}$. 6

c. Let $A=\{1,2,3,4,5\}\times\{1,2,3,4,5\}$ and Define R on A by $(x_1,y_1)R(x_2,y_2)$ if $x_1+y_1=x_2+y_2$, 8

(i) Determine the equivalence classes $\{(1,3),(2,4)\ and\ (1,1)\}$

(ii) Determine the partition of A induced by R.

UNIT - V

9 a. Define the following with an example,

(i) Homomorphism

(ii) Isomorphism 10

(iii) Cyclic groups.

b. Prove that identity element and inverse of every element is unique in a group G. 10

10 a. An encoding function $E:Z_2^2\rightarrow Z_2^5$ is given by the Generator matrix,

$$G=\begin{bmatrix}1 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & 1 & 1\end{bmatrix}$$

(i) Determine all code words. What can be said about the errors detection capability of this code? 10

(ii) What about its error correction capability?

(iii) Find the associated parity check matrix H.

b. Write a short note on:

(i) Parity check 10

(ii) Group codes.

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