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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, B.E. - Semester End Examination; Dec - 2016/Jan - 2017

Engineering Mathematics - I

(Common to all Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find the n^{th} derivative of, i) $e^{2x}\cos^3 x$ ii) $\log_{10}\left\{(1-2x)^3(8x+1)^5\right\}$. 6

b. Find the n^{th} derivative of, $\dfrac{6x}{(x^2-4)(x-1)}$. 7

c. If $y=e^{m\cos^{-1}x}$, Prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$. 7

2 a. State Lagrange's mean value theorem, verify Lagrange's mean value theorem for the function $f(x)=2x^2-7x+10$ in [2, 5]. 6

b. State Cauchy's mean value theorem, verify Cauchy's mean value theorem for the function $f(x)=\sqrt{x}$ and $g(x)=\dfrac{1}{\sqrt{x}}$ in $[a,b]$. 7

c. Expand $\log(1+e^x)$ in ascending powers of x upto the term containing x^4. 7

UNIT - II

3 a. Evaluate: i) $\lim\limits_{x\to 0}\dfrac{e^{2x}-(1+x)^2}{x\log(1+x)}$ ii) $\lim\limits_{x\to 0}\left[\dfrac{\tan x}{x}\right]^{1/x^2}$. 6

b. If $\lim\limits_{x\to 0}\dfrac{x[1-a\cos x]+b\sin x}{x^3}=\dfrac{1}{3}$ find a and b. 7

c. Find the angle of intersection between the curves $r=\dfrac{a\theta}{1+\theta}$ and $r=\dfrac{a}{1+\theta^2}$. 7

4 a. Find the Pedal equation of the curve $r^m=a^m(\cos m\theta+\sin m\theta)$. 6

b. Find the radius of curvature for the curve $x=a\left[\cos t+\log\tan\left(\dfrac{t}{2}\right)\right]$, $y=a\sin t$ at any point t. 7

c. Show that for the curve $r^n=a^n\cos n\theta$ the radius of curvature is $\dfrac{a^n}{(n+1)r^{n-1}}$. 7

UNIT - III

5 a. If $u=x^2\tan^{-1}\left(\dfrac{y}{x}\right)-y^2\tan^{-1}\left(x/y\right)$, Show that $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{x^2-y^2}{x^2+y^2}$. 6

b. If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, Show that $x^2\dfrac{\partial^2 u}{\partial x^2}+y^2\dfrac{\partial^2 u}{dy^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}=\sin 4u-\sin 2u$. 7

c. If $u=f\left(x-y,y-z,z-x\right)$ find, $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}$. 7

6 a. Find the components of velocity and acceleration at t = 2 on the curve, $\vec{r}=\left(t^2+1\right)\hat{i}+\left(4t-3\right)\hat{j}+\left(2t^2-6t\right)\hat{k}$ in the direction of $\hat{i}+2\hat{j}+2\hat{k}$. 6

b. Find the directional derivative of $\phi=\dfrac{xz}{x^2+y^2}$ at (1, -1, -1) in the direction of $\vec{A}=\hat{i}-2\hat{j}+\hat{k}$. 7

c. Find the value of the constant 'a' such that $\vec{F}=\left(axy-z^3\right)\hat{i}+\left(a-2\right)x^2\hat{j}+\left(1-a\right)xz^2\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F}=\nabla\phi$. 7

UNIT - IV

7 a. Obtain the reduction formula for $\int\sin^n x dx$ and hence evaluate $\int_0^{\pi/2}\sin^n x dx$, where n is the positive integer. 6

b. Evaluate $\int_0^{\pi}\dfrac{\sin^4\theta}{\left(1+\cos\theta\right)^2}$ using reduction formula. 7

c. Trace the curve: $y^2\left(a-x\right)=x^3,\ \ a>0$. 7

8 a. Find the length of an arch the cycloid $x=a(\theta-\sin\theta),\ \ y=a(1-\cos\theta)$. 6

b. Find the perimeter of the curve $r=a(1+\cos\theta)$. 7

c. Show that: $\int_0^{\infty}e^{-x^2}\cos\alpha x dx=\dfrac{\sqrt{\pi}}{2}e^{-\alpha^2/4}$ by differentiating under the integral sign. 7

UNIT - V

9 a. Solve: $\left(x-4y-9\right)dx+\left(4x+y-2\right)dy=0$. 6

b. Solve: $y\left(2x-y+1\right)dx+x\left(3x-4y+3\right)dy=0$. 7

c. Solve: $\left(y\log_e x-2\right)ydx=xdy$. 7

10 a. Solve: $\left(1+e^{x/y}\right)dx+e^{x/y}\left[1-\dfrac{x}{y}\right]dy=0$. 6

b. Find the orthogonal trajectory of the family $r^n\cos n\theta=a^n$. 7

c. If the air is maintained at 30ºC and the temperature of the body cools from 80ºC to 60ºC in 12 minutes, find the temperature of the body after 24 minutes. 7

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