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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester, Master of Computer Applications (MCA)

Make-up Examination; Feb - 2017

Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. How many 8 character password can be constructed selecting 5 alphabets followed by 3 digits such that,

i) Without any restriction ii) Without repetition of a alphabets and digits

iii) Only vowels must be used. 6

b. How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7, if we want n to exceed 5,000,000 or even. 7

c. Find the number of ways in which 7 apples and 6 oranges are distributed among 5 children such that each child must get atleast one apple. 7

2 a. For any three sets A B C verifies.

$A\Delta(B\Delta C) = (A\Delta B)\Delta C$ 6

b. Determine $|A \cup B \cup C|$ when

$|A| = 50, \quad |B| = 500, \quad |C| = 5000$

i) $A \subseteq B \subseteq C$ ii) $A \cap B = B \cap C = A \cap C = \phi$

iii) $|A \cap B| = |A \cap C| = |B \cap C| = 3$ and $|A \cap B \cap C| = 1$ 7

c. When two fair dice are rolled what is the probability that;

i) 6 is the sum of two dice. ii) Sum is at least 7. iii) Sum is even. 7

UNIT - II

3 a. Define tautology and show that, $\left[(p \vee q) \wedge (p \to r) \wedge (q \to r)\right] \to r$ is a tautology. 6

b. Prove the logical equivalence without using truth table.

$(p \to q) \wedge (\neg q \wedge (r \vee \neg q)) \approx q \wedge p$ 6

c. Express symbolically and check validity of the given argument.

"If it rains, I'll not come to your house

If I come to your house, we will go for shopping.

It is raining.

Therefore, we will not go for shopping. 8

Contd…..2

4 a. Define quantifiers with two examples for each. 6

b. If $p(m): x \geq 0$ $r(x): x^2 - 3x - 4 = 0$

$q(x): x^2 \geq 0$ $s(x): x^2 - 3 > 0$

Find the truth value of the following, 8

i) $xp(x) \wedge q(x)$ ii) $\forall xp(x) \rightarrow q(x)$ iii) $\forall xr(x) \vee s(x)$ iv) $xp(x) \wedge r(x)$

c. Show that the argument is valid.

"No engineering student of first or second semester studies logic. Anil is an engineering student who studies logic. $\therefore$ Anil is not in second semester. 6

UNIT - III

5 a. Write a direct and indirect proof for the statement "If n is odd then n + 9 is even" 6

b. If $H_1 = 1$, $H_2 = 1 + 1/2$, $H_3 = 1 + 1/2 + 1/3 \ldots H_n = 1 + 1/2 + \ldots + 1/n$

are Harmonic numbers then $\forall n \in Z^+$, $\sum_{i=1}^{n} H_i = (n+1)H_n - n$ 7

c. Obtain the recursive definition for the sequence in each of the following,

i) $a_n = 5n$ ii) $b_n = 2 - (-1)^n$ iii) $a_n = 9n + 8$ 7

6 a. Define stirlings number of second kind and evaluate S(8, 6). 7

b. Let f, g be two functions defined on Z, as

$f(x) = 2x + 1$, $g(x) = x^3 - x$ 7

find $fog(x), gof(x), fof(x), gog(x), f^{-1}(x), g^{-1}(x)$

c. State pigeonhole principle. An office employs 13 clerks. Show that at least 2 of them will have birthdays during the same month of the year. 6

UNIT - IV

7 a. Let 'R' be a relation defined as "exactly divides on A = {1, 2, 3, 6, 20, 50, 80}

i) Write M (R) ii) Prove that R is a partially ordered relation. 8

iii) Draw the Hasse diagram of (A,R)

b. i) How many relations are there from A to B if |A| = 5 |B| = 4

ii) How many binary relations are there on A

iii) How many binary relations are there on B.

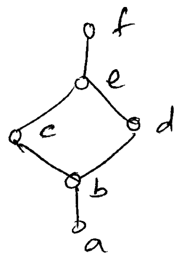
iv) How many binary relations on A are reflexive relations? 6

v) How many binary relations on B are reflexive relations?

vi) How many are equivalence relation on A.

Contd…..3

c. Define LUB, GLB of a subset B = {c, d, e} of A whose Hasse diagram is given below. 6



8 a. Let A = {1, 2, 3, 4, 5} and define R on A x A by (x, y), R(x_2, y_2) iff $x_1 + y_1 = x_2 + y_2$

i) Verify that R is an equivalence relation on A x A 8

ii) Determine the partition induced by R on A x A.

b. Let A = {1, 2, 3, 4} B = {w, x, y, z} C = {5, 6, 7}

$R_1 : A \rightarrow B$ defined by R_1 = {(1, x) (2, x) (3, y) (3, z)}

$R_2 : B \rightarrow C$ defined by R_2 = {(w, 5) (x, 6)} 6

$R_3 : B \rightarrow C$ defined by R_3 = {(x, 5) (w, 6)}

Find (R_1 o R_2), (R_1 o R_3), (R_1 o R_2) $\cup$ (R_1 o R_3), (R_1 o R_2) $\cap$ (R_1 o R_3)

c. Draw the Hasse diagram that represents positive divisors of 50, 100. 6

UNIT - V

9 a. Distinguish between:

i) Simple and multiple graphs ii) Connected and disconnected graphs 8

iii) Euler graphs and Hamiltonian graphs.

b. Define isomorphism between two graphs with an example. 6

c. Write short notes on Konigsberg bridge problem related to origin of graph theory. 6

10 a. Define; i) Rooted tree, ii) Binary tree

iii) M – ary tree iv) Complete m – ary tree 6

v) Balanced tree with an example for each.

b. Construct an optimal prefix code tree for the message “HAPPY JOURNEY”. Indicate the code that has been generated by the tree. 7

c. Find the minimal spanning tree of the given connected graph using,

i) Prim’s Algorithm ii) Krushkal’s Algorithm 7

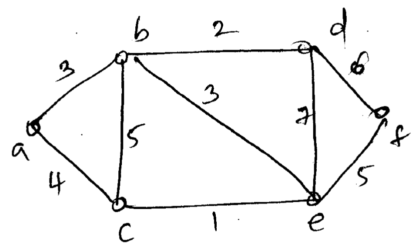


fig 10c

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