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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Computer Science and Engineering
Semester End Examination; May/June - 2018
Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Define Isomorphism of graphs. Show that the graphs shown in Fig. Q1(a) are isomorphic to each other. 6

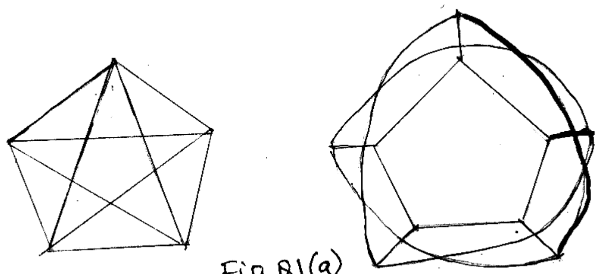


Fig. Q1(a)

b. Prove that a simple graph with '*n*' vertices and '*k*' components can have at most *(n-k) (n-k+1)/2* edges. 7

c. How many vertices will the following sub graphs have, if they contain :
i) 16 edges and all vertices are of degree 4 3
ii) 21 edges, 3 vertices of degree 4 and other vertices of degree 3

d. Discuss Konigsberg bridge problem. 4

2 a. Define: i) Regular graph ii) Self complementary graph iii) Euler circuit. Give one example for each. 6

b. Prove that in the complete graph with '*n*' vertices, where '*n*' is an odd number ≥ 3, there are *(n-1)/2* edge-disjoint Hamiltonian cycles. 6

c. Prove that a connected graph G has an Euler circuit (i.e. G is an Euler graph) if and only if G can be decomposed into edge-disjoint cycles. 8

UNIT - II

3 a. Define: i) Self dual graph ii) Bipartite graph iii) Chromatic number. Give one example for each. 6

b. If G is a connected simple planar graph with *n* (≥3) vertices, *e* (>2) edges and *r* regions, then prove that, i) $3r \leq 2e$ ii) $e \leq 3n - 6$. 7

c. (i) Find the dual graph for the planar graph shown in Fig. Q3(c). Write down any four observations of the graph given and its dual. 4

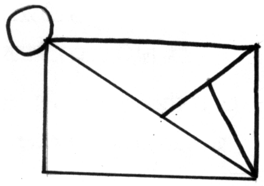


Fig. Q3(c)

(ii) Verify the Euler's formula for the Fig. Q3(c). 3

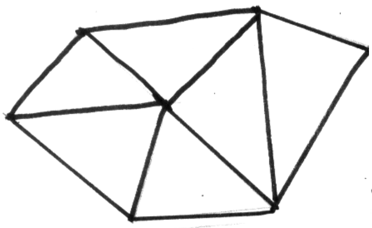


Fig. Q3(c)

4 a. Show that the complete graph *K*5 is a non-planar graph. 5

b. Find the chromatic polynomial for the following graph Fig. 4(b). Hence, find the chromatic number. 6

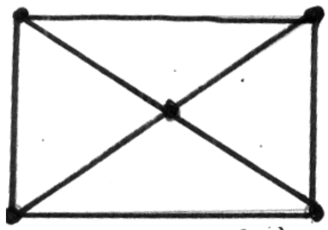


Fig. Q4(b)

c. Define Planar graph and also check the planarity for the graph given in Fig. Q4(c) by the elementary reduction method. 5

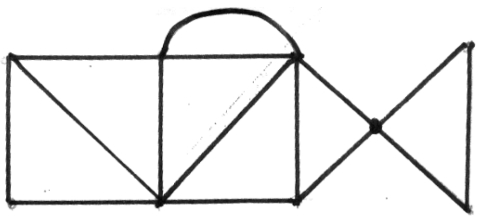


Fig. Q4(c)

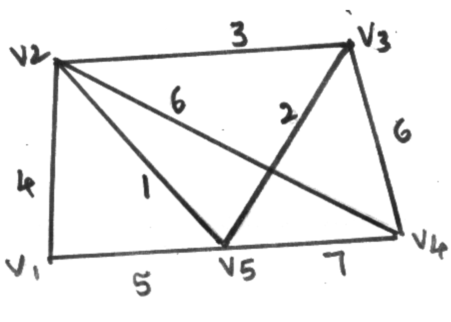
d. Show that the bipartite graphs $K_{2,2}$ and $K_{2,3}$ are planar graphs. 4

UNIT - III

5 a. For every tree with $|V| \geq 2$, show that tree has atleast 2 pendant vertices. 7

b. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown in Fig. Q5(b). 6

Fig. Q5(b)



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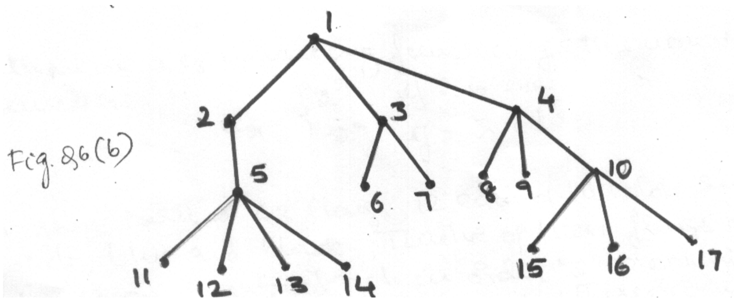
c. Construct an optimal prefix code for the symbols *a, b, c, d, e, f, g* that occur (in a given sample) with respective frequencies 10, 30, 5, 15, 20, 15, 5. 7

6 a. i) Define a tree. In every tree T = (V, E). Show that $|V| = |E| + 1$

ii) If a tree has four vertices of degree 2 one vertex of degree 3, two of degree 4, and one of degree 5. How many pendant vertices does it have? 8

b. List the vertices in the tree shown in Fig. Q6(b) when they are visited in pre-order traversal and in a post-order traversal? 6



c. Explain max-flow mincut theorem. Apply this to network shown in Fig. Q6(c) to obtain max-flow between *A* and *Z*. 6

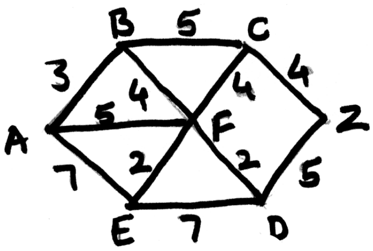


Fig.Q6(c)

UNIT - IV

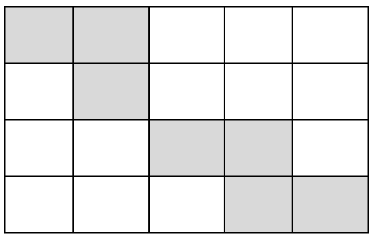
7 a. How many integers between 1 and 300 (inclusive) are, 7

i) Divisible by atleast one of 5, 6, 8? ii) Divisible by none of 5, 6, 8?

b. Find the number of derangements of integers from 1 to 2*n* (inclusive) satisfying the condition that the elements in the first *n* places are: 6

i) 1, 2, 3,……. *n* in some order ii) *n+1, n+2,* …..*2n* in some order.

c. Obtain the Rook polynomial of the shaded blocks of the chess board shown in Fig. Q7(c). 7



8 a. Find a generating function for the following sequence 1, 1, 0, 1, 1, 1,……. 6

b. Determine a generating function for the numeric function; 6

$$a^r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

c. A ship carries 48 flags, 12 each of the colors red, white, blue and black. Twelve of there flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and an odd no of black flags? 8

UNIT - V

9 a. Solve the recurrence relation $a_n - 3a_{n-1} = 5(7^n)$ for $n \geq 1$, given that $a_0 = 2$. 7

b. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$, given that $a_1 = 5$, $a_2 = 3$. 6

c. Solve the recurrence relation: $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given $F_0 = 0$, $F_1 = 1$. 7

10 a. Solve the recurrence relation: 8

$2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, for $n \geq 0$, with $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.

b. Solve the recurrence relation: 7

$a_n + a_{n-1} - 8a_{n-2} - 12a_{n-3} = 0$ for $n \geq 3$, with $a_0 = 1$, $a_1 = 5$, $a_2 = 1$.

c. If a_n is a solution of the recurrence relation $a_{n+1} = ka_n$ for $n \geq 0$, and $a_3 = 153/49$ and $a_5 = 1377/2401$, what is k? 5

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