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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Information Science and Engineering

Semester End Examination; May/June - 2018

Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

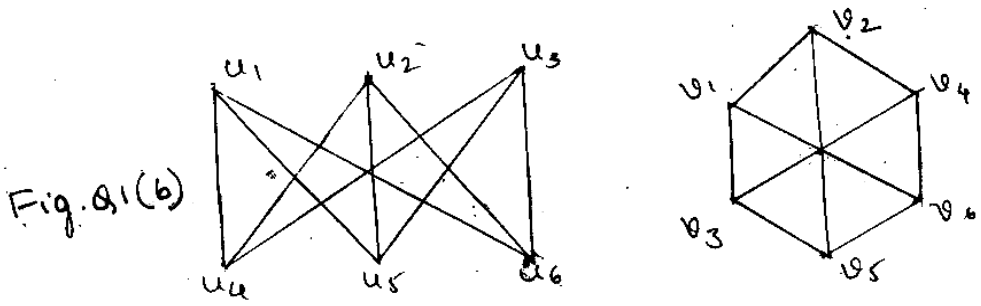
***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Define the following with one example for each : 6

i) Complete graph ii) Induced subgraph iii) Complete bipartite graph

b. Define isomorphism of graphs. Show that the graphs given in Fig. Q1(b) are isomorphic. 6



c. Prove that a connected graph G has an Euler circuit (i.e, G is an Euler graph) if and only if all vertices of G are even degree. 8

2 a. How many vertices will the following subgraph have? If they contain : 6

i) 16 edges and all vertices are degree 4

ii) 21 edges, 3 vertices of degree 4 and other vertices of degree 3

b. Prove that in any undirected graph, the number of odd degree vertices is even. 6

c. Give an example for; 8

i) A graph which has an Euler circuit but no Hamilton cycle

ii) A graph which has a Hamilton cycle but no Euler circuit

iii) A graph has a Hamilton path but no Hamilton circuit

iv) Self complementary graph with your vertices

UNIT - II

3 a. Define a Planar graph. Show that the completely bipartite graph $K_{3,3}$ is non planar. 6

b. Show that a connected planar graph G with '*v*' vertices and '*e*' edges has exactly *e-v+2* regions in every one of its diagrams. 6

c. What is the length of the longest path in each of the following graphs : 4

i) $K_{m,n}$ where m, $n \in Z^+$ and $m < n$ ii) $K_{2,4}$

iii) $K_{3,8}$ iv) $K_{8,14}$

Contd…2

d. Find the dual graph for the planar graph shown in Fig. Q3(d) and verify whether the given graph is self dual or not. 4

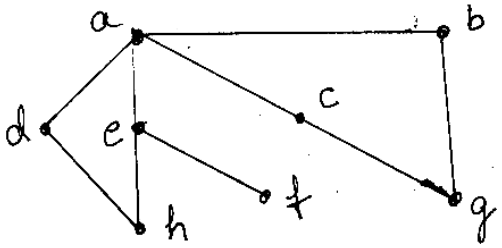


Fig. Q3(d)

4 a. By using the method of elementary reduction, show that the graph given in Fig. Q 4(a) is planar. 6

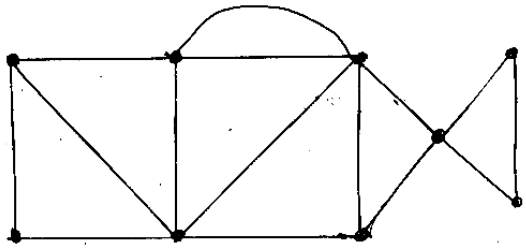


Fig. Q4(a)

b. Define chromatic number of a graph. Find the number of polynomial for the graph shown in the Fig. Q4(b) and also find the chromatic number for the same. 7

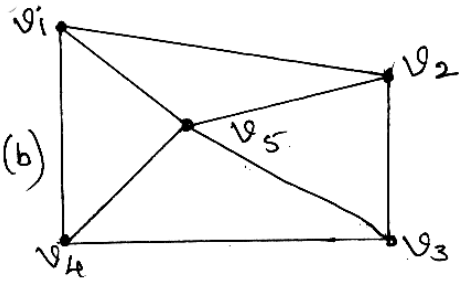


Fig. Q4(b)

c. Consider the graph $K_{2,3}$ shown in Fig. Q4(c). Let λ denote the number of colors available to properly color the vertices of this graph. Find; 7

i) How many proper colorings of the graph have vertices a, b, colored same

ii) How many colorings of the graph have vertices a, b, colored with different colors?

iii) The chromatic polynomial of the graph

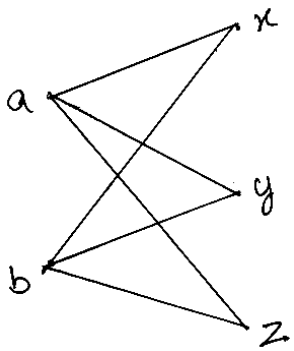


Fig. Q4(c)

UNIT - III

5 a. Define the following with example for each : 6

i) Binary Rooted tree ii) Prefix code iii) Balanced tree

Contd...3

b. Prove that if G = (V, E) is an undirected graph and then G is connected, if and only if G has a spanning tree. 6

c. Obtain an optimal prefix code for the message "FALL OF THE WALL". Indicate the code. 8

6 a. i) Let $T_1 = (V_1, E_1)$, $T_2 = (V_2, E_2)$ between trees where $|E_1| = 19$ and $|V_2| = 3|V_1|$. Determine;

$|V_1|, |V_2|$ and $|E_2|$ 6

ii) What is the sum of degrees of all the vertices in the tree with 2000 vertices?

b. State the Kruskal's algorithm. Using Kruskal's algorithm find the minimal spanning tree for the weighted graph shown in Fig. Q 6(b).

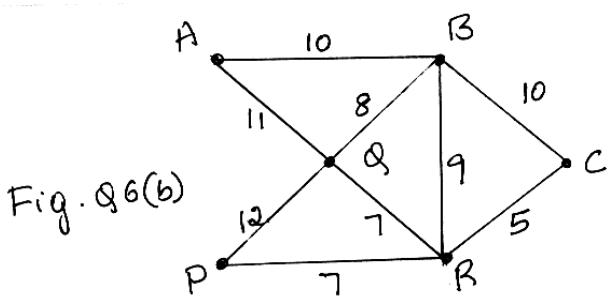


Fig. Q6(b) 7

c. Apply Dijkstra's algorithm for the weighted graph G = (V, E) shown in Fig. Q 6(c) to find shortest distance from V_1 to each of the other vertices in the graph. Indicate the weights of these shortest paths.

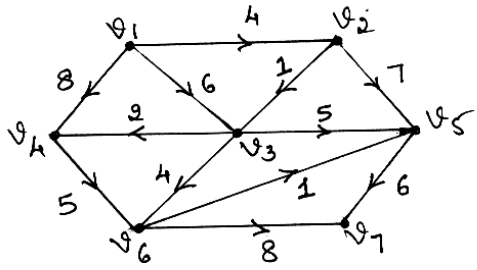


Fig. Q6(c) 7

UNIT - IV

7 a. Determine the number of positive integers '*n*' where $1 \leq n \leq 100$ and '*n*' is not divisible by 2, 3 or 5. 7

b. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to the right person. 6

c. Obtain the rook polynomial for the board shown in Fig. Q7(c) using the expansion formula.

		1
	2	3
4	5	6
7	8	

7

Fig. Q7(c)

8 a. Determine the coefficient of x^{15} in $f(x) = (x^2+x^3+x^4+\ldots..)^4$ 7

Contd…4

b. Determine the sequence generated by $(1-4x)^{-\frac{1}{2}}$. 6

c. A ship carries 48 flags 12 each of the colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and an odd number of black flags? 7

UNIT - V

9 a. A bank pays 6 % (annual) interest on savings, compounding the interest monthly. If Bonnie deposits $1000 on the first day of May, how much will this deposit be worth a year later? 6

b. Solve the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2};\ a_0 = 2, a_1 = 5$ 7

c. Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200;\ n \geq 0, a_0 = 3000, a_1 = 3300$. 7

10 a. Find the generating function for the recurrence relation :
$C_n = 3C_{n-1} - 2C_{n-2},\quad n \geq 2$ and $C_1 = 5, C_2 = 3$. 7

b. Using generating function solve;
$Y_{n+2} - 4y_{n+1} + 3y_n = 0$, given $y_0 = 2, y_1 = 4$. 7

c. The number of virus affected files in a system is 1000 and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. 6

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