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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; May/June - 2018

Engineering Mathematics - IV

(Common to AU, CV, ME & IP Branches)

Time: 3 hrs. *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Show that $w = z + e^z$ is analytic and hence find $\frac{dw}{dz}$. 6

b. Find analytic function, whose real part is $u(x, y) = x^4 + y^4 - 6x^2y^2$. 7

c. Discuss the transformation $w = z + \frac{1}{z},\ z \neq 0$. 7

2 a. Evaluate $\int_c |z|\, dz$, where C is the cantour

i) Straight line from $z = -i\ to\ i$ 6

ii) Left half of the unit circle $|z| = 1$ from $z = -i\ to\ z = i$

b. Expand $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in a region $1 < z+1 < 3$ using Laurent's series. 7

c. Evaluate $\int_c \frac{e^z}{\cos \pi z}\, dz$, where C is the unit circle $|z| = 1$, using Cauchy's residue theorem. 7

UNIT - II

3 a. Find the root of the equation $xe^x = \cos x$ using the Regula falsi method in [0, 1] correct to four decimal places. 6

b. Using Newton-Raphson's method approximate the root of $3x = \cos x + 1$ in [0, 1] upto four decimal places. 7

c. Find the real root of the equation $x^3 - x - 1 = 0$ using fixed point iteration method. Accelerate the convergence by Aitken's Δ^2 - method. Correct to three decimals. 7

4 a. Use Taylor's series method to find $y(0.1)$ and $y(0.2)$ from $\frac{dy}{dx} = x^2y - 1,\ y(0) = 1$. 6

b. Using Runge-Kutta method of fourth order to find $y(0.2)$ from $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$. and step size $h = 0.2$ 7

c. Apply Milne's method to compute $y(1.4)$ given that $\frac{dy}{dx} = x^2 + \frac{y}{2}$ with $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. (Use correcter formula twice). 7

Contd…2

UNIT - III

5 a. The first four moments about an arbitrary value 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Find the skewness and kurtosis. 6

b. If V and R are related by a relation of the type $R = a + bV^2$. Find by the method of least squares with the help of the table 7

V:	10	20	30	40	50
R:	8	10	15	21	30

c. If θ is the angle between the lines of regression. Show that $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$. 7

6 a. A random variable $(X = x)$ has the following probability distribution for values of x 6

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find;

i) k ii) Evaluate $P(x \geq 6)$ and $P(3 < x \leq 6)$

b. In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisons distribution to calculate the appropriate number of packets containing i) No defective ii) One defective iii) Two defective blades in consignment of 10,000 packets. 7

c. In a test of 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life 2040 Hrs and standard deviation of 60 Hrs. Estimate the number of bulbs likely to burn for i) More than 2150 Hrs ii) Less than 1950 Hrs (Given $\phi(1.83) = 0.4664$ and $\phi(1.5) = 0.4332$). 7

UNIT - IV

7 a. The joint distribution of two random variables X and Y is as given below. 6

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find the marginal distribution of X and Y. Also find $E(X)$, $E(Y)$ and $E(XY)$ and $cov(X, Y)$.

b. Find the fixed probability vector of the regular stochastic matrix 7

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

c. If the joint probability function for (x, y) is $f(x,y)=\begin{cases} C(x^2+y^2), & 0\le x\le 1, \quad 0\le y\le 1, c\ge 0 \\ 0, & otherwise \end{cases}$

Determine the following : 7

i) Value of constant 'C'

ii) Marginal density functions of x and y

iii) $P\left(x<\frac{1}{2}, y>\frac{1}{2}\right)$

iv) $P\left(\frac{1}{4}<x<\frac{3}{4}\right)$

8 a. Solve the equations: $10x-2y-3z=205$, $-2x+10y-2z=154$, $-2x-y+10z=120$ by Relaxation method. 6

b. Solve the system of equations: $2x+y+4z=12$, $8x-3y+2z=20$, $4x+11y-z=33$ using Gauss-Seidal method with initial conditions (1, 0, 1). (Perform three iterations). 7

c. Determine the largest Eigen value and the corresponding Eigen vector of the matrix with initial Eigen vector $[1, 0, 1]^T$ using Power method. (Perform three iterations). 7

$$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

UNIT - V

9 a. Derive an Euler's equation. 6

b. Find the extermal of the functional $\int_0^{\pi/2}\left(y'^2-y^2+2xy\right)dx$ with $y(0)=0$ and $y\left(\frac{\pi}{2}\right)=0.$ 7

c. Define geodesic. Find the geodesic on a surface given that the arc length on the surface is 7

$$S=\int_{x_1}^{x_2} x\sqrt{1+(y')^2}\ dx.$$

10 a. Obtain the series solution of $\frac{d^2y}{dx^2}+xy=0.$ 6

b. Obtain series solution of the Bassel's differential equation $x^2y''+xy'+\left(x^2-n^2\right)y=0.$ 7

c. Express $f(x)=x^4+3x^3-x^2+5x-2$ in terms of Legendre's polynomials. 7

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