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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; May/June - 2018

Engineering Mathematics - IV

(Common to EE, EC, CS, IS Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Find a real root of the equation $x\log_{10} x = 1.2$ correct to five decimal places using the Newton Raphson method. 6

b. Find real root of the equation $\cos x = 3x - 1$ correct to three decimal places using Regula falsi method. 7

c. Find a real root of the equation $x^3 - x - 1 = 0$ using fixed point iteration method. Accelerate the convergence by Aitkin's Δ^2-method. Carry out three iterations. 7

2 a. Using modified Euler's method find y at $x = 0.2$ given that,

$\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. Perform three iterations at each step. 6

b. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation

$\frac{dy}{dx} = \frac{y - x}{y + x}$ with $y(0) = 1$ at taking $h = 0.1$. 7

c. Apply Adams-Bashforth method to solve the equation; $(y^2 + 1)dy - x^2 dx = 0$ at $x = 1$ given $y(0) = 1$, $y(0.25) = 1.0026$, $y(0.5) = 1.0206$, $y(0.75) = 1.0679$. Applying corrector formula twice. 7

UNIT - II

3 a. Define Vector space and Subspace. Give with suitable examples. 6

b. Find the change of basis matrix P from usual basis,

$E = \{e_1, e_2, e_3\}$ of R^3 to the basis $S = \{w_1 = (1, 1, 1), w_2 = (1, 1, 0), w_3 = (1, 0, 0)\}$. 7

c. Find Rank and nullity of the linear transformation

$T : R^3 \to R^3$ by $T(x, y, z) = (x + z,\ x + y + 2z,\ 2x + y + 3z)$. 7

4 a. Apply Gauss-Seidal iterative method to solve the equations

$27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$. Perform three iterations. 6

b. Solve by relaxation method: $10x - 2y - 2z = 6$; $-x + 10y - 2z = 7$; $-x - y + 10z = 8$. 7

c. Determine the largest eigen value and the corresponding eigen vector of the matrix 7

$A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. Taking initially $[1,\ 0,\ 0]^T$. (Perform six iterations).

UNIT - III

5 a. Show that $f(z)=\sin z$ is analytic and hence find $f'(z)$. 6

b. Find analytic function $f(z)$ as a function of z given that the sum of its real and imaginary part is $x^3-y^3+3xy(x-y)$. 7

c. Find the Bilinear transformation that maps the points $z_1=0$, $z_2=-i$, $z_3=2i$ into the points $w_1=5i$, $w_2=\infty$, $w_3=-i/3$ respectively. 7

6 a. Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the line $x=2y$. 6

b. Expand $f(z)=\dfrac{1}{(z-1)(2-z)}$ as a Laurent's series valid for (i) $|z|<1$ (ii) $1<|z|<2$. 7

c. Using the Cauchy's residue theorem, evaluate $\int_c \dfrac{Sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}$ where C is the circle $|z|=3$. 7

UNIT - IV

7 a. In a certain distribution $\{x_i, f_i\}_{i=1,2,3,\ldots n}$. The first four moments about the point '5' are -1.5, 17, -30 and 108. Calculate the skewness and kurtosis. 6

b. Fit a parabola for the following data : 7

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

c. If θ is the angle between the two regression lines, show that $\tan\theta=\left(\dfrac{1-r^2}{r}\right)\dfrac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$. 7

Explain the significance when $r=0$ and $r=\pm1$.

8 a. The probability distribution of a finite random variable x is given by the following table : 6

x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	k	0.2	$2k$	0.3	k

Find the value of k, mean and variance.

b. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each microsecond. Compute the probabilities of; 7

(i) No error during a micro second (ii) One error per microsecond

(iii) At least one error per microsecond

Contd…3

c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D. of the distribution [$\phi(0.5) = 0.19$; $\phi(1.4) = 0.42$]. 7

UNIT - V

9 a. The joint distribution of two random variables X and Y is as follows: 6

$X \backslash Y$	-4	2	7
1	$1/8$	$1/4$	$1/8$
5	$1/4$	$1/8$	$1/8$

Find ;

(i) $E(X)$ and $E(Y)$ (ii) $E(XY)$ (iii) σ_x and σ_y

b. Find the unique fixed probability vector for the regular stochastic matrix, 7

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

c. If X and Y are contineous random variables having the joint density function

$$f(x, y) = \begin{cases} C\left(x^2 + y^2\right); & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

Determine; 7

(i) Constant 'C'

(ii) $P\left(X < \frac{1}{2}, Y > \frac{1}{2}\right)$

(iii) $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$

10 a. Obtain the series solution of the differential equation $\frac{d^2y}{dx^2} + xy = 0.$ 6

b. Obtain $J_n(x)$ as a solution of the Bassel's differential equation $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0.$ 7

c. State Rodrigue's formula. Express $x^3 + x^2 + x + 1$ in terms of Legendre's polynomials. 7

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