Estimation and Guidance for Manoeuvring Targets

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ABSTRACT: The rigorous development in the field of highly manoeuvrable threats gives a huge platform to find the efficient integrated estimation and guidance algorithms. In the present day technology, various kinds of threats are getting advanced day by day. For example ballistic missiles with time varying parameters or low flying cruise missile. Successful intercepting these kinds of highly manoeuvrable targets demands high performance at the end phase. Achieving zero miss distance is a challenge in case of highly manoeuvrable targets. In this paper one of the efficient estimation technique Kalman filtering and guidance algorithm, two dimensional Proportional Navigation law are discussed. Future work of this research is detailed.

Keywords: Miss distance, Kalman Filtering, Proportional Navigation guidance law.

I. INTRODUCTION

The natural process of improvement of all aspects of our lives includes advances in the development of sophisticated weapon systems, the means to defend ourselves from enemies and those who consider wars as a way to improve their living conditions. The basic goal of defence is to destroy the target which never changes. The vast development in the field of defence is never ending. The result of this is the development of highly manoeuvrability in enemy targets such as air crafts, missiles, ships and submarines. This is the major challenge for defence section to track them accurately and to intercept them. Accepting, intercepting manoeuvring targets has been a major challenge to the guided missile community, much attention is given to development of efficient estimation and guidance algorithms. Due to manoeuvrability of the targets the miss distances of few meters are considered admissible. Successful interception of anti-surface missiles carrying lethal war heads are more dangerous requires a very less miss distance or even a direct hit. Hence there is goal to alleviate this problem by developing an optimal estimation technique and the guidance laws.

The rest of the paper is organized as follows: section II is about problem formulation section III is about major estimation technique which is required to track the targets with either linear motion or nonlinear motion and discusses guidance laws in section IV. Section V gives the simulation results section VI gives conclusion and future research work.

II PROBLEM FORMULATION

Successful interception of maneuvering targets is far more difficult compared to non-maneuvering targets. Two scenarios like intercepting ballistic missile with time varying parameters and low flying cruise missiles requires a great effort to design an efficient integrated estimation and guidance algorithm. As a basic start to this research, a scenario is considered where target's motion is maneuverable. Sample is taken every 0.1 sec. Intercepting target with the above specifications is achieved using Kalman filtering technique and classical guidance law. Further research work is based on a scenario with low flying, quick moving targets like cruise missile, an effort to design an integrated estimation and guidance algorithm for the mentioned scenario using hybrid filters and optimal guidance laws.

III ESTIMATION AND GUIDANCE

A. Kalman Filter

The Kalman filter is a classical efficient optimal filter for targets having linear motion and if the errors are Gaussian. Early identification of the correct target manoeuvre model will allow smaller estimation errors and consequently smaller miss distances. [1] The Kalman filter is a tool that can estimate the variables of a wide range of processes. In mathematical terms we would say that a Kalman filter estimates the states of a linear system. The Kalman filter not only works well in practice, but it is theoretically attractive because it can be shown that of all possible filters, it is the one that minimizes the variance of the estimation error. The Kalman filter is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms. It estimates the state of dynamic system. This dynamic system can be disturbed by some noise, mostly assumed as white noise. To improve the estimated state the Kalman filter uses measurements that are related to the state but disturbed as well. [2]

Thus the Kalman filter consists of two steps:

1. The prediction
2. The correction.

Kalman filter can be summarized as below:

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$$\mathbf{K_k=P'_kH^T(H\,P'_kH^T+R)^{-1}} \qquad (1)$$

Equation (1) is the Kalman gain equation

The state update is given by

$$\mathbf{\hat{x}_k=\hat{x}'_k+K_k(z_k-H\,\hat{x}')} \qquad (2)$$

this is the update equation for the new estimate, combining the old estimate with the measurement data and the kalman gain. The covariances can be updated using the equation

$$\mathbf{P_k=(I-K_kH)P'_k} \qquad (3)$$

this is the update equation for the error covariance matrix with the optimal gain. This is the filtered covariance which is find out after the processing of the state estimates.

$$\mathbf{\hat{x}'_{k+1}=\Phi\,\hat{x}_k} \qquad (4)$$

$$\mathbf{P_{k+1}=\Phi\,P_k\Phi^T+Q} \qquad (5)$$

This equations are the state update and the predicted covariance matrix equations. Predicted covariance is the set of errors before processing the state estimates. The Q and R in the above equatins are the process noise and the measurement noise covariances respectively. x_k is the state vector of the process at time k(n×1), Φ is the state transition matrix of the process from the state at k to the state k+1,and assumed stationery over time,(n×m), H is the noiseless connection between the state vector and the measurement vector. The objective of this filter is to minimize the mean squared error between the actual and the estimated data. It provides the best estimate of the data in the mean squared error sense.

Q must be initialized when we initialize the filter. And this covariance is updated with time. For a single dimension it is given by equation 6:

$$Q=2\alpha\sigma_m^2\begin{bmatrix} T^5/20 & T^4/8 & T^3/6 \\ T^4/8 & T^3/3 & T^2/2 \\ T^3/6 & T^2/2 & T \end{bmatrix} \qquad (6)$$

Where σ_m^2 is the variance of the target acceleration and α is the reciprocal of the maneuver (acceleration) time constant

$$\sigma_m^2=A^2_{max}/3[1+4P_{max}-P_0]$$

For a three dimensional model the Q (9×9 matrix) is multiplied with the identity matrix as shown in equation 7:

$$Q=2\alpha\sigma_m^2\begin{bmatrix} I(T^5/20) & I(T^4/8) & I(T^3/6) \\ I(T^4/8) & I(T^3/3) & I(T^2/2) \\ I(T^3/6) & I(T^2/2) & I(T) \end{bmatrix} \qquad (7)$$

Pk for three dimensional filter is 9×9 matrix and Pk can be initialized as shown below as in equation 8:

$$Pk=\begin{bmatrix} [R] & [RV] & [RA] \\ [VR] & [V] & [VA] \\ [AR] & [AV] & [A] \end{bmatrix} \qquad (8)$$

Where R is measurement noise covariance and given by equation 9:

$$R=\begin{bmatrix} \sigma r\hat{}2 & 0 & 0 \\ 0 & r\hat{}2\sigma\theta\hat{}2 & 0 \\ 0 & 0 & r\hat{}2\sigma\varphi\hat{}2 \end{bmatrix} \qquad (9)$$

The state transition matrix Φ is the transition matrix from state k to state k+1 and is given by equation 10:

$$\Phi=\begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

For the three dimensional model Φ it is multiplied with the identity matrix to result in 9×9 matrix as in equation 11.

$$\Phi=\begin{bmatrix} I(1) & I(T) & I(T^2/2) \\ 0 & I(1) & I(T) \\ 0 & 0 & I(1) \end{bmatrix} \qquad (11)$$

Kalman filter is an optimal filter for targets with linear motion. But if we consider targets which are highly manoeuvrable the performance of Kalman filters is poor. In a guided missile system the estimator performs dual tasks: the task of a filter (smoothing and averaging the noise-corrupted. measurements) and the task of an observer (reconstructing the nonmeasured or nonmeasurable state variables, such as target acceleration). Satisfactory performance of each task requires a different type of estimator design[5]. And also targets possess different motions. Sometimes they move in linear motion sometimes they manoeuvre. Hence for different target motions a hybrid filter is optimal. [7]

The IMM estimator is a suboptimal hybrid filter and one of the most cost effective hybrid state estimation schemes.[8] Each cycle of IMM estimator consists of three major steps: interaction (mixing), filtering, and combination. The algorithm requires multiple filters, each corresponding to the target's acceleration state[7]. Since it uses multiple models to model different target motions it gives optimal estimates of the target states.

IV GUIDANCE

A. Proportional Navigation

A missile guidance system is a group of components that measures the position of the guided missile with respect to its target and changes the missile flight path on accordance with a guidance law. It includes sensing computing and control components. [3]

A guidance law is defined as an algorithm that determines the required commanded missile acceleration. The goal of guidance is to reach a target. The basic guidance law we can name is the classical proportional navigation guidance law. It issues acceleration commands, perpendicular to the instantaneous missile target line of sight, which are proportional to the line of sight rate and closing velocity. . Mathematically, the guidance law can be stated as in equation 12 [4]

$$n_c = N' \, V_c \, \lambda dot \qquad (12)$$

where n_c is the acceleration command(in ft/s^2), N' a unit less designer chosen gain(usually in the range of 3 to 5) known as effective navigation ratio, V_c is the missile target closing velocity in ft/s and λ is the line of sight angle (in rad). λ_{dot} is the time derivative of the line of sight angle.

Figure 1 shows the two dimensional missile target engagement geometry

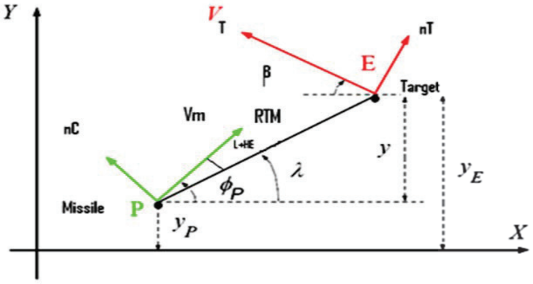

Fig. 1. Two dimensional missile target engagement geometry

As the future enhancement in this research, it deals with differential game based guidance laws. In the design of DGL law it is assumed that missile demanded acceleration can be instantaneously switched from one value to another. However, toggling of the control at a very fast rate will result in the problem commonly called chattering. The next work of this research is concentrating more on chatter removal algorithms

V SIMULATION RESULTS

A. Estimation Results:

The fig 2 is plot of the true position, measured position and estimated position values of a vehicle travelling in straight line. The x axis is the time in seconds and the y axis is the position in feet. Blue line indicates the true and the green is measured and the red is the estimated value. The commanded acceleration is constant 1 m/sec^2. The position measured 10 times per second. There is 2 to 3 meter of error between the true and estimated positions.

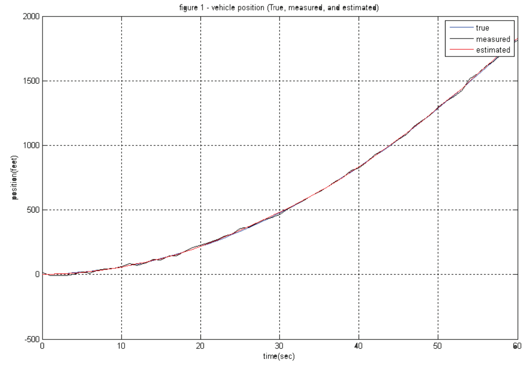

Fig. 2. Plot of the true position, measured position and estimated position values

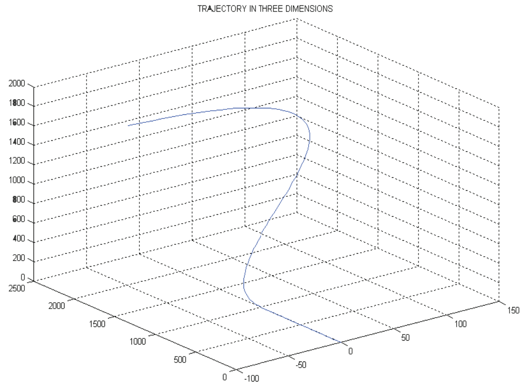

Fig. 3.Three dimensional trajectory

The above plot fig 3 shows a three dimensional trajectory. The time duration is up to the trajectory generated is about 120 seconds. The above trajectory is plotted without adding the noise. The simulation input is the velocity equal to 300 km/hr. The samples are taken for every 0.1 second.

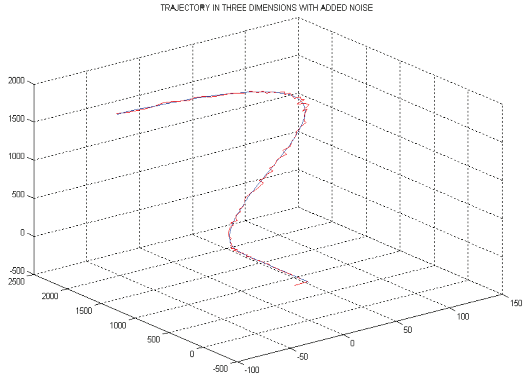

Fig. 4.3D Trajectory with the added noise

The above plot is the 3D trajectory with the added noise. The noise is added is random noise. The blue line is the trajectory and red line is the trajectory with added noise.

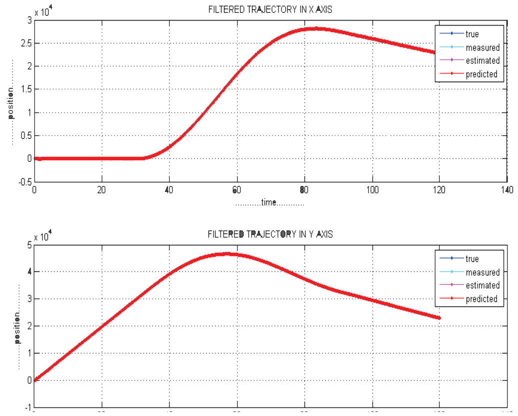

Fig. 5. Filtered trajectory in x and y directions

The figure 5 shows the plot of the filtered trajectory in x and y directions. The various components like the true, measured, estimated and predicted value are plotted**.** Here samples are taken for every 0.1 second for more accurate reading**.**

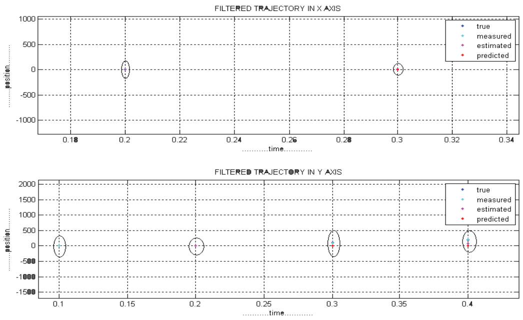

Fig. 6. Beginning points of the track

The above plot fig 6 shows the beginning points of the track. Initially two starting positions of the trajectory is given as the simulated input. From the third point the filter starts estimating the future values considering previous values. Blue dots are true values, cyan dots are the measured values, magenta is for the estimated and red is for the predicted values.

B. Proportional Navigation Results

For the final interception we use the proportional Navigation algorithm. The simulation inputs are the initial location of the missile and target, speeds, and effective navigation ratio. The error sources can be varied are target manoeuvre and heading error. In this section few results are discussed.

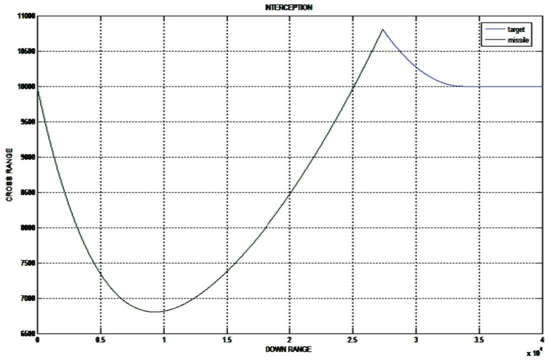

Fig. 7.Interception with heading error -40 degree

In the figure 7 the target travelling at 1000m/sec^2 and the missile is at 3000 m/sec^2. The heading error is -40 degree and the effective navigation ratio is N =5. Hence missile initially takes off with this heading error as shown in the plot. Later on it is guided towards target by the help acceleration commands generated for the missile. As this parameter decreases the removal of the heading error delays. The acceleration magnitude of the target is varied to 40 keeping the condition when range is below 20km. the acceleration magnitude is varied to make the target manoeuvre. Down range is nothing the horizontal distance travelled by the missile and cross range is the vertical distance

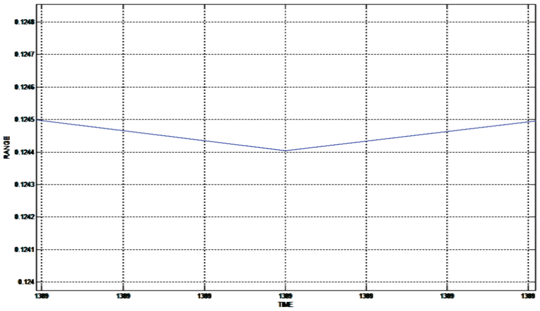

Fig. 8.Enlarged diagram

The figure 8 is the range plot of figure 7. The miss distance in this case is 0.12 which is approximately equal to zero which results in the successful interception of the target. In this section heading error has been varied from -70 to +70. The table I show the heading error range and the corresponding miss distances obtained. The source of errors we can vary is the heading error and the other one is acceleration magnitude. In this set of results, acceleration magnitude is kept constant, assuming the target is not maneuvering.

Table I. Miss distance versus heading error

HEADING ERROR (IN DEGREES)	MISS DISTANCE (IN METERS)
-70	0.05
-60	2.8
-50	1.2
-40	2.5
-30	2.7
-20	0.4
-10	2.5
0	0
+10	2.5
+20	0.4
+30	2.7
+40	2.5
+50	1.2
+60	2.8
+70	0.05

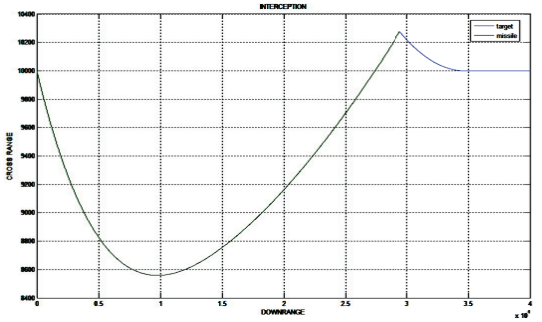

Fig. 9.Interception with heading error -20 and acceleration magnitude 20

The fig 9 shows the interception at acceleration magnitude 20 and heading error -20. The acceleration magnitude is changed at the condition when range is less than 20 km.

The table II below shows the varying miss distance with respect to acceleration magnitude. In this case we are keeping the heading error as constant. The table 2 below shows the list of values of acceleration magnitude and corresponding miss distances.

Table II. Acceleration magnitude and miss distance

ACCELERATION MAGNITUDE (nt)	HEADING ERROR	MISS DISTANCE
10	-20	2.8
20	-20	1.0
30	-20	1.5
40	-20	3.2
50	-20	1.6
60	-20	1.2
70	-20	2.7

V CONCLUSION AND FUTURE WORK

This paper presents the basic work of estimation and guidance, which includes simulation results of intercepting a low rate(1000m/sec^2) manoeuvring target. This paper includes design of basic Kalman filter and two dimensional proportional navigation guidance law. Future work is designing efficient integrated estimation and guidance laws for low flying cruise missiles, quick reaction missiles using hybrid filters and optimal guidance laws. Based on above mentioned points the next work will be designing two dimensional IMM filter and DGL law. The work will later be extended to three dimensional IMM filter and DGL law along with chatter removal algorithm for seeker based interceptors.

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