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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Computer Science and Engineering

Semester End Examination; May/June - 2019

Graph Theory and Combinatorics

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

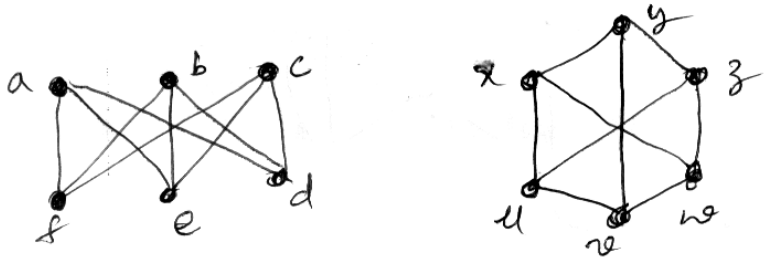
UNIT - I

1 a. Define the following with example : 8

i) Complete graph ii) Connected graph

iii) Euler graph iv) Hamiltonian graph

b. Define Isomorphism and show that the following graphs are isomorphic : 6



c. Prove that a complete graph of *n* vertices has $\frac{n(n-1)}{2}$ edges using MIP. 6

2 a. Prove that a connected graph G has an Euler circuit, if and only if all vertices of G are of even degree. 8

b. Explain Konigsberg Bridge problem related to a graph. 6

c. Draw the following graphs : 6

i) Euler and Hamiltonian ii) Not Euler and Hamiltonian

iii) Euler and Not Hamiltonian iv) Not Euler and Not Hamiltonian

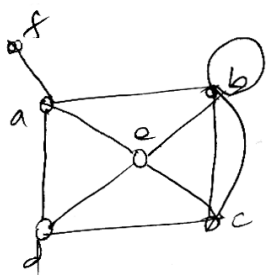
UNIT - II

3 a. Distinguish between planar and non-planar graphs with an example. Prove that $K_{3,3}$ is non-planar. 8

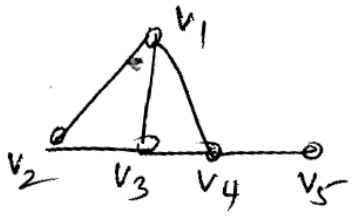
b. Prove that "Every connected planar graph with *n* vertices and *e* edges has $e - n + 2$ regions (i.e., $f = e - n + 2$) 7

c. Write the steps involved in detection of planarity. 5

4 a. Write self dual graphs. Construct the dual of the given graph, 8



b. Define chromatic number and chromatic polynomial and find the same of the graph given below, 7



c. State Decomposition theorem and Multiplication theorem. 5

UNIT - III

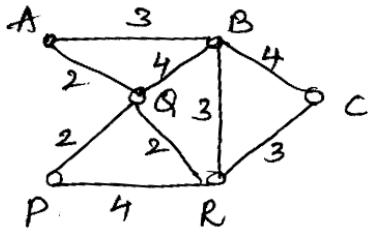
5 a. List six properties of a tree graph. 6

b. Define directed, rooted, m-ary, binary complete m-ary trees. 7

c. Constructed an optimal prefix code tree and its code for the symbols, *a*, *o*, *q*, *u*, *y*, *z* that occurs with frequencies 20, 28, 4, 17, 12, 7 respectively. 7

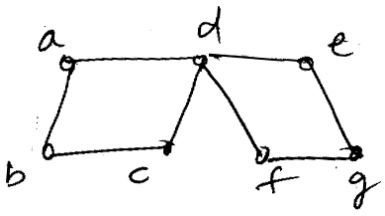
6 a. Define spanning tree and construct minimal spanning tree by,

i) Kruskal's algorithm ii) Prism's algorithm



10

b. Find BFS and DFS spanning trees for the graphs given below starting from vertex "*a*". 5



c. Write the steps involved in Dijkstra's algorithm. 5

UNIT - IV

7 a. In how many ways 5A's, 4B's, 3C's can be arranged so that all the identical letters are not in a single block? 6

b. In how many ways can the integers be arranged such that no object is in its original position? Derive d_n. 7

c. Find the Rook polynomial of the board given below : 7

1	2			
	3			
		4	5	
			6	7

8 a. Find the sequences generated by the following functions : 6

$i) \frac{x^2}{1-x}$ $ii) 3x^3 + e^{5x}$ $iii) \frac{1}{1-x^2}$

b. Find the coefficient of x^{18} in the following products : 7

$(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + \ldots\ldots)^5$

c. A company appoints 11 software engineers each of whom is to one of four officers of the company. Each office should get atleast one of these engineers. In how many ways can these assignments be made? 7

UNIT - V

9 a. The number of virus affected files in a system is 1000 and this increases 150% every two hours. Use recurrence relation to determine the number of virus affected files in the system after one day. 6

b. Solve: $a_n - 6a_{n-1} + 9a_{n-2} = 0 \ \forall n \geq 2$ given that $a_0 = 5, a_1 = 12$. 7

c. Solve: $2a_{n+3} = a_{n+2} + 29_{n+1} - a_n \ \forall n \geq 0, a_0 = 0, a_1 = 1, a_2 = 2$. 7

10 a. Solve the recurrence relation : 6

$F_{n+2} = F_{n+1} + F_n \ \forall \ n \geq 0$ and $F_0 = 0, F_1 = 1$.

b. Solve: $a_{n+2} + 3a_{n+1} + 2a_n = 3^n \ \forall n \geq 0$ and $a_0 = 0, a_1 = 1$. 7

c. Using the generating function method solve the recurrence relation, 7

$a_n - 3a_{n-1} = n \ \forall n \geq 1$ given $a_0 = 1$.

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