P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Information Science and Engineering

Semester End Examination; May/June - 2019

Graph Theory and Combinatorics

*Time: 3 hrs*   *Max. Marks: 100*

Note: Answer **FIVE** full questions, selecting **ONE** full question from each unit.
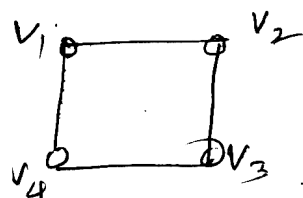
UNIT - I

1 a. Define the following with an example for each :

   i) Null graph and Complete graph   ii) Regular graph and Simple graph   6

   iii) Connected and Disconnected graph

   b. Define Isomorphism between two graphs and;

   i) Draw two graphs that are isomorphic

   ii) Draw two graphs that are not isomorphic but have same number of vertices and same number   7

   of edges

   c. Define Bipartite graph. Prove that "Every circuit in a Bipartite graph has even number of edges".   7

2 a. Prove that "Every connected graph is Euler iff degree of every vertex in it must be even.   6

   b. Determine $|V|$ for the following graphs :   7

   i) Regular with 15 edges   ii) 10 edges and 2 vertices of degree 4 remaining vertices of degree 3

   c. Draw the following graphs :   7

   i) Euler and Hamiltonian   ii) Euler and not Hamiltonian   iii) Not Euler and not Hamiltonian

UNIT - II

3 a. Define planar graph. Prove that $K_{3,3}$ is non planar.   6

   b. Define; i) Self complementary graph   ii) Self dual graph with an example for each.   7

   c. Prove that a connected graph of *n* vertices and *e* edges has $e - n + 2$ regions.   7

4 a. List the properties between the graph and its dual and draw the dual of the given graph,


   8
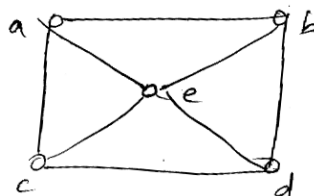
   b. How to detect planarity in a given graph.   6

   c. Define chromatic number and chromatic polynomial of a graph and find the same of the graph

   given below

   6

UNIT - III

5 a. List all properties of a tree (atleast 6).   6

  b. Obtain an optional prefix code tree and code for the message "HAPPY BIRTHDAY".   7

  c. Define the following :   7

   i) Rooted tree   ii) *m*-ary tree   iii) Complete *m*-ary tree   iv) Balanced tree

6 a. Prove that every tree is minimally connected and two chromatic.   6

  b. Using Prim's algorithm, find the minimal spanning tree of the graph given below :   7
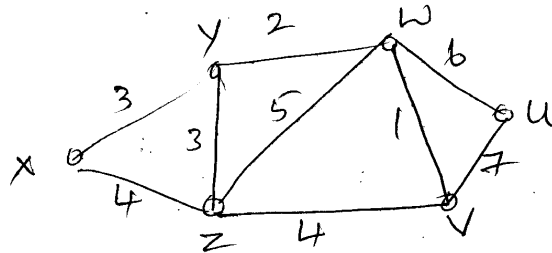
  c. Explain the steps involved in Dijkstra's algorithm.   7

UNIT - IV

7 a. Define derangement. Find the number of derangements of 1 to 10 in a single line such that no even number is in its original position.   6

  b. Find the number of permutations of the English letters which contains;   7

   i) Exactly two   ii) Atleast two of the patterns CAR, DOG, PUN and BYTE

  c. Define Rook's polynomial. Find the Rook's polynomial of 3×3 board.   7

8 a. Find the generating function for each of the following sequences :   8

   i) 1, 1, 1, 0, 0, 0, ……………   ii) 1, 2, 3, 4, 5, 6, …………………..

   iii) $1^3, 2^3, 3^3, 4^3$,……………………   iv) 1, −2, 3, −4, 5, −6,…………………….

  b. Find the sequence of the generating function; *i)* $2x^2(1-x)^{-1}$ *ii)* $2x^3 + \frac{1}{1-x}$ 5

  c. A ship carries 48 flags, 12 each of the colors red, white, blue and black. 12 of these flags are placed on vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and an odd number of black flags.   7

UNIT - V

9 a. Find the unique solution of the recurrence relation, $6a_n - 7a_{n-1} = 0$ $n \geq 1$ $a_3 = 343$.   6

  b. Solve the recurrence relation;   7

   $F_{n+2} = F_{n+1} + F_n$ $n \geq 0$ and $F_0 = 0, F_1 = 1$.

  c. Solve the recurrence relation;   7

   $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$ $n \geq 0$ $a_0 = 0, a_1 = 1, a_2 = 2$.

10 a. Solve $a_n - 3a_{n-1} = n$ $n \geq 1, a_0 = 1$ by method of generating functions.   8

  b. Solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$ $a_0 = 3, a_1 = 7$.   7

  c. A bank pays a certain % of annual income on deposits compounding the interest in 3 months. If the amount doubles in 6 yrs and 6 months, what is the annual % of interest paid by the bank?   5

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