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P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Fourth Semester, B.E. - Semester End Examination; May / June - 2019

Engineering Mathematics - IV

(Common to EE, EC, CS&E, IS&E Branches)

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. Using Bisection method, find a approximate root of the equation $\sin x = \frac{1}{x}$ that lies between $x = 1$ and $x = 1.5$ (measured in radians). Carryout computations up to the fourth stage. 6

b. Find a real root of the equation $\cos x = xe^x$ using the Regula-Falsi method. Perform three iterations. 7

c. Using Newton–Raphson method, find the real root of $x \log_{10} x = 1.2$. Perform four iterations. 7

2 a. Using modified Euler's method find $y(0.2)$ correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2, y(0) = 1$ taking $h = 0.1$. 6

b. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ taking $h = 0.1$. 7

c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \left(\frac{y}{2}\right)$ and the following data: $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. 7

UNIT - II

3 a. Define a vector space with a suitable example. 6

b. Find the dimension and basis of the subspace spanned by the vectors $(2, -3, 1), (3, 0, 1), (0, 2, 1)$ and $(1, 1, 1)$ of $V_3(R)$. 7

c. Find linear transformation. $T : V_2(R) \to V_2(R)$ such that $T(1, 2) = (3, 0)$ and $T(2, 1) = (1, 2)$. 7

4 a. Apply Gauss-Seidel iteration method to solve the system of equations: $x + y + 54z = 110$; $27x + 6y - z = 85$; $6x + 15y + 2z = 72$. Obtain the solution up to an accuracy of three decimal places by carrying three iterations. 6

b. Using Relaxation method, solve the system of equations: $12x + y + z = 31$, $2x + 8y - z = 24$, $3x + 4y + 10z = 58$ 7

Contd…2

c. Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method, taking the initial Eigen vector [1, 1, 1]′. Carryout five iterations. 7

UNIT - III

5 a. Show that $f(z)=\sin z$ is analytic. Hence find its derivative. 6

b. Construct the analytic function $f(z)$ whose real part $u=e^{-x}\left[\left(x^2-y^2\right)\cos y+2xy\sin y\right]$. 7

c. Discuss the transformation $W=e^z$. 7

6 a. Evaluate $\int_C \bar{Z}\, dz$ where C represents the following paths : 6

i) Straight line from $-i$ to i

ii) The right half of the unit circle |Z| = 1 from $-i$ to i

b. Expand $f(z)=\dfrac{2z+3}{(z+1)(z-2)}$ in a Laurent's series valid for the regions: 7

i) |z| < 1 ii) 1 < |z| < 2 iii) |z| > 2

c. Using Cauchy's Residue theorem, evaluate: 7

$$\oint_C \frac{\sin \pi z^2+\cos \pi z^2}{(z-1)^2(z-2)}\,dz \quad \text{where C : } |z| = 3.$$

UNIT - IV

7 a. Compute skewness and kurtosis if the first four moments of the frequency distribution of *f(x)* about the value $x = 4$ are respectively 1, 4, 10 and 45. 6

b. Fit a straight line of the form $y = ax + b$ for the data given below using the method of least squares: 7

x	5	10	15	20	25
y	16	19	23	26	30

c. Obtain the lines of regression and hence find the coefficient of correlation for the data: 7

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

8 a. A random variable x has the following probability function for various values of x 6

x	0	1	2	3	4	5	6	7
P(x)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find k ii) Evaluate P(x < 6), P ($x \geq 6$), P ($3 < x \leq 6$)

Contd…3

b. The probability that a pen manufactured by a factory to be defective is $\frac{1}{10}$. If 12 such pens are manufactured what is the probability that; 7

i) Exactly 2 are defective ii) At least 2 are defective iii) None of them are defective

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be: 7

i) Less than 65 ii) More than 75 iii) Between 65 and 75

Given ϕ (1) = 0.3413.

UNIT - V

9 a. Using two random variables X and Y as follows: 6

X \ Y	0	1	2	3
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

i) Determine the distributions of X and Y

ii) Determine the joint distribution of X and Y

iii) Calculate $E(X)$ and $E(Y)$

b. Define the following : 7

i) Probability vector

ii) Stochastic matrix

iii) Regular stochastic matrix

iv) Fixed probability vector.

c. Show that the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. 7

Also determine the associated unique fixed probability vector.

10 a. Obtain a series solution of $y'' - xy' + y = 0$. 6

b. Prove that: 7

i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

c. Express $f(x) = x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. 7

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