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# P.E.S. College of Engineering, Mandya - 571 401

***(An Autonomous Institution affiliated to VTU, Belagavi)***

**Fourth Semester, B.E. - Semester End Examination; May / June - 2019**

**Engineering Mathematics - IV**

**(Common to AU, CV, ME & IP Branches)**

*Time: 3 hrs* *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

**UNIT - I**

1 a. Show that $f(z) = \cos z$ is analytic. Hence find its derivative. 6

b. Find an analytic function whose imaginary part is $v(x, y) = e^x\left\{(x^2 - y^2)\cos y - 2xy\sin y\right\}$. 7

c. Discuss the conformal transformation $W = Z + \frac{1}{Z}, \quad Z \neq 0$. 7

2 a. Verify Cauchy's theorem for the integral of $f(z) = \frac{1}{z}$ taken over the triangle formed by the points (1, 2), (3, 2), and (1, 4). 6

b. Expand $f(z) = \dfrac{z^2 - 1}{z^2 + 5z + 6}$ as Laurent's series in the regions; i) 2 < |z| < 3 ii) $|z| > 3$ 7

c. Using the Cauchy's residue theorem evaluate, $\displaystyle\int_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)^2(z-2)}\, dz$ 7

Where C is the circle $|z| = 3$.

**UNIT - II**

3 a. By using Bisection method, obtain an approximate root of the equation $\sin x = \frac{1}{x}$ that lies in $\left(1, \frac{1}{2}\right)$ (perform four iterations). 6

b. Obtain an approximate real root of $x\log_{10} x = 1.2$ correct to four decimal places using Newton-Rapson method that lies in (2, 3). 7

c. Find the root of the equation $x^3 - 5x + 1$ near $x = 0.5$ using fixed point iteration and accelerate the convergence by Aitken's $\Delta^2$ – method. 7

4 a. Find $y(1.1)$ from Taylor's series method up to third degree, if $y(x)$ satisfies $\dfrac{dy}{dx} = \log_e(xy),\ y(1) = 2.$. 6

b. Using modified Euler's method, find $y$ at $x$ = 0.4 given $\dfrac{dy}{dx} = y + e^x$, $y(0)$ = 0 taking $h$ = 0.2. Perform three iterations at each step. 7

c. Apply Adams-Bash forth method to compute $y(0.4)$ given $\dfrac{dy}{dx} = (x - y^2),\ y(0) = 1$ given that $y(0.1) = 0.9117,\ y(0.2) = 0.8494$ and $y(0.3) = 0.8061$. 7

Contd…2

## UNIT - III

5 a. The first four moments about an arbitrary value 28.5 of frequency distribution are 0.294, 7.144, 42.409 and 454.98. Find the skewness and kurtosis based on moments. 6

b. The pressure and volume of a gas are related by $PV^{\gamma} = K$, γ and K being constants. Fit this equation using 7

| P: | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
|---|---|---|---|---|---|---|
| V: | 1.62 | 1.00 | 0.75 | 0.62 | 0.52 | 0.46 |

c. Obtain the correlation coefficient and regression lines from the data, 7

| $x$ | 21 | 23 | 30 | 54 | 57 | 58 | 72 | 78 | 87 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 60 | 71 | 72 | 83 | 110 | 84 | 100 | 92 | 113 | 135 |

6 a. The probability density function of a variable $X$ is, 6

| *X:* | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| *P(X):* | *K* | *3K* | *5K* | *7K* | *9K* | *11K* | *13K* |

Find $K$, $P(X < 4)$, $P(X \geq 5)$, $P(3 < X \leq 6)$.

b. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets. 7

c. In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation, given $\phi(0.5) = 0.19$ and $\phi(1.4) = 0.42$. 7

## UNIT - IV

7 a. The joint probability function for two discrete random variables $X$ and $Y$ is given by $f(x,y) = c(2x+y)$ where $x$ and $y$ are integral values such that $0 \leq x \leq 2$, $0 \leq y \leq 3$ and $f(x,y) = 0$ otherwise. Find; i) Constant C ii) $P(X = 2, Y = 1)$ iii) $P(X \geq 1, Y \leq 2)$ 6

b. The joint distribution of two random variables X and Y is given by, 7

| X \ Y | 1 | 3 | 9 |
|---|---|---|---|
| 2 | $\frac{1}{8}$ | $\frac{1}{24}$ | $\frac{1}{12}$ |
| 4 | $\frac{1}{4}$ | $\frac{1}{4}$ | 0 |
| 6 | $\frac{1}{8}$ | $\frac{1}{24}$ | $\frac{1}{12}$ |

Find marginal distributions of $X$ and $Y$ and evaluate COV($X$, $Y$) and e($X$, $Y$).

c. Define; i) Stochastic matrix (ii) Regular stochastic matrix.

Find the fixed probability vector of the regular stochastic matrix: $A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ 7

Contd…3

8 a. Solve by Gauss-Seidal method, the equation:

$3x+20y-z=-18,\ 20x+y-2z=17,\ 2x-3y+20z=25$ 6

with initial approximation (*x*, *y*, *z*) = (0, 0, 0) (Perform three iterations).

b. Solve by Relaxation method: $5x+2y+z=12,\ x+4y+2z=15,\ x+2y+5z=20.$ 7

c. Find the largest Eigen value and the corresponding Eigen vector of A = $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by taking initial vector as $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ using power method (Perform four iterations). 7

**UNIT - V**

9 a. Find the extremal of the functional $\int_0^{\pi/2} \left[ (y')^2 - y^2 + 4y\cos x \right] dx,\ y(0) = y(\tfrac{\pi}{2}) = 0.$ 6

b. Find the geodesics on a right circular cylinder of radius "*a*". 7

c. Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. 7

10 a. Obtain the series solution of the equation $\dfrac{d^2y}{dx^2} - y = 0.$ 6

b. Solve the Bessel's differential equation $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0.$ 7

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. 7

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