U.S.N | | | | | | | | | | |

P.E.S. College of Engineering, Mandya - 571 401

(An Autonomous Institution affiliated to VTU, Belagavi)

Third Semester, B.E. - Computer Science and Engineering
Semester End Examination; Dec. - 2019
Discrete Mathematical Structures

Time: 3 hrs *Max. Marks: 100*

***Note**: Answer **FIVE** full questions, selecting **ONE** full question from each unit.*

UNIT - I

1 a. In a high school, a teacher must select nine students from 28 juniors and 25 seniors for a volleyball team;
(i) In how many ways can this be done?
(ii) In how many ways this is done if 2 juniors and one senior who are best spikers and must there in the team?
(iii) If the team must have four juniors and five seniors — 6

b. Define the term combinations with repetition and find the number of +ve integer solutions of the equation, $x_1 + x_2 + x_3 + x_4 + x_5 < 10 \ \ where \ \ x \geq 0$. — 7

c. Find the coefficient of $x_1^{n_1} x_2^{n_2} .....x_k^{n_k}$ in $\left(x_1 + x_2 + x_3 + ...x_k\right)^n$ where $n = n_1 + n_2 + ... + n_k$ and find the coefficient of $w^3 x^2 y\, z^2$ in $\left(2w - x + 3y - 2z\right)^8$ — 7

2 a. Simplify: (i) $\overline{\left[\overline{(A \cup B) \cap C}\right] \cup \overline{B}}$ using laws.
(ii) $\overline{A \Delta B} = A \Delta \overline{B} = \overline{A} \Delta B$ using membership table method. — 6

b. Let U = set of all three digit integers and A_i represent integers in U where i is in i^{th} place. Compute; i) $\left|A_1 \cup A_2 \cup A_3\right|$ ii) $\left|A_1 - \left(A_2 \cup A_3\right)\right|$ iii) $\left|\overline{A_1 \cup A_2}\right|$ — 7

c. i) If a dice is rolled three times what is the probability that all throws have same value?
ii) When a fair coin is tossed five times what is the probability that at least one head appears up? — 7

UNIT - II

3 a. Define; i) Tautology ii) Logical equivalence iii) Converse, inverse, contra positive statements. — 6

b. Simplify using laws of logic theory,
i) $\left[\neg(q \to r) \wedge r\right] \wedge (p \to q)$ ii) $\left[(p \vee \neg q) \wedge (\neg p \vee \neg q)\right] \vee q$ — 7

c. "Raju's car keys are in his bag or they are on the school table.
Raju's car keys are not on the school table.
Therefore Raju's keys are in his bag."
Express in the symbolic form and check validity of the argument. — 7

4 a. Represent the statement using quantifiers and also negate each: 6

i) Every integer is not divisible by 5

ii) Some triangles are right angled triangle

iii) At least one integer is even

b. State; i) Rule of universal specification 7

ii) Rule of existential specification

iii) Rule of universal Generalization.

c. Express (for $x \in \mathbb{R}$ = Universe) the argument and check validity. 7

" *If* $3x-7=20$ then $3x=27$.

if $3x=27$ then $x=9$.

Therefore if $3x-7=20$ then $x=9$.

UNIT - III

5 a. Prove by Mathematical Induction Principle $4n < n^2 - 7 \;\; \forall\, n \geq 6$. 6

b. Prove by all three methods. For every integers "If *n* is odd then *n + 7* is even". 7

c. If L_n represents Lucas number and F_n represents Fibonacci number. Prove that, 7

$\forall_n \in Z^+ \quad L_n = F_{n-1} + F_{n+1}$ (Lucas number $L_n = L_{n-1} + L_{n-2} \forall n \geq 2$ where $L_0 = 2,\ L_1 = 1$

6 a. Define: i) One-One functions ii) Onto functions, with example for each. 6

b. Find the range of the following functions: 7

$g : \mathbb{R} \to \mathbb{R}\, g(x) = x^2$

$f : Z^+ \to \mathbb{R}\;\; f(x) = \frac{1}{x}$ excluding zero.

$h : Z \times Z \to Z \quad h(x, y) = 2x + 3y$

c. i) Define *S(m, n)*, find *S(3, 3), S(3, 4).* 7

ii) If *f*(*x*) = *2x + 5* a function on $\mathbb{R}$. Prove that *f* is invertible function and find the inverse function of *f*.

UNIT - IV

7 a. i) Define a Matrix and Digraph of a binary relation on A. 7

ii) Find the conditions that represent reflexivity, symmetry, transitivity in the matrix form.

b. Let "*R*" be relation defined as (*a, b*) ∈ *R* 7

iff "*a* + *b* = even" on *A* = {1, 3, 4, 7}

i) Prove that *R* is an equivalence relation (Justify)

ii) Find the partition induced by *R* on A.

c. How many equivalence relations are there on A with m elements (explain). 6

Contd…3

8 a. Draw Hasse diagram representing positive divisors of 15, 30, 50. 7

b. Define; i) Partially ordered set ii) Totally ordered set iii) Lattice. 7

c. If S, R are two relation defined as "exactly divides" and "a + b > 3" on A = {1, 2, 3, 4}. Find S, R, S°R, R°S, S^2, R^2. 6

UNIT - V

9 a. Define a group and give an example of a group and not a group. 6

b. i) Show that any group G is an Abelian group iff $(ab)^2 = a^2b^2 \forall a,b \in G$

ii) State Lagrange's theorem. 7

c. Let $f:(R^+,\times) \to (R,+)$

Where $f(x) = \log_{10} x$ 7

i) Prove that *f* is both one–one and onto ii) *f* is an isomorphism.

10a. Write short notes on;

i) Encoding ii) Decoding iii) Haming metric 6

b. Define the encoding function: $E: Z_2^3 \to Z_2^6$ by means of the parity–check matrix.

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$ 8

i) Determine all code words,

ii) Does this code correct all single errors in transmission?

c. Prove that in a group code, the minimum distance between the distinct code word is the minimum of the weight of the non zero elements of the code. 6

* * *