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## P.E.S. College of Engineering, Mandya - 571401

(An Autonomous Institution affiliated to VTU, Belagavi)
Third Semester, B.E. - Computer Science and Engineering Semester End Examination; Dec. - 2019

Discrete Mathematical Structures
Time: 3 hrs
Max. Marks: 100
Note: i) PART - A is compulsory. Two marks for each question.
ii) PART - B: Answer any Two sub questions (from $a, b, c$ ) for Maximum of 18 marks from each unit.
Q. No. Questions ..... Marks
I : PART - A ..... 10
I a. Define the rules of sum and product. ..... 2
b. Define the principle of duality. ..... 2
c. State Pigeon-Hole principle. ..... 2
d. Define lattice with example. ..... 2
e. Brief the term cyclic groups. ..... 2
II : PART - B ..... 90
UNIT - I ..... 18
1 a. Find the number of license plates created which contains two English alphabets followed by four digits, i) with repetition and ii) without repetition.
Determine the coefficient of $x^{2} y^{2} z^{3}$ in the expansion of $(3 x-2 y-4 z)^{7}$.
b. The board of directors of pharmaceutical corporation has 10 members. An upcoming stock holders meeting is scheduled to approve a new state of company officers (chosen from the board members)
i) How many different states consisting of president, vice president, secretary and treasurer can be board present to the stock holders for their approval?
ii) Three members of the board of directors are physicians. How many states from part (i) have,
I) A physician nominated for presidency
II) Exactly one physician appearing on the slate
c. A manufacturer of 2000 automobile batteries is concerned about defective terminals and defective plates. If 1920 of batteries have neither defect, 60 have defective plates and 20 have both defects. How many batteries have defective terminals?
UNIT - II
2 a. i) Prove the following logical equivalence:

$$
(p \rightarrow q) \wedge[\neg q \vee(r \vee \neg q) \equiv(q \wedge p)]
$$

ii) Convert the following statement to symbolic form and also write its negation "for all $x$, if $x$ is odd then $x^{2}-1$ is even".
b. Find whether the following argument is valid;

If a triangle has two equal sides, then it is isosceles.
If a triangle is isosceles, then it has two equal angles.
The triangle ABC does not have two equal angles.
Therefore ABC does not have two equal sides.
c. Prove that the following argument is valid where in ' $c$ ' is specified element of the universe:

$$
\begin{aligned}
& \forall x[p(x) \rightarrow q(x)] \\
& \forall x[q(x)-r(x)] \\
& \neg r(c) \\
& \therefore \neg p(c)
\end{aligned}
$$

## UNIT - III

3 a. Prove by mathematical induction that $1^{2}+3^{2}+5^{2}+------+(2 n-1)^{2}=1 / 3 n(2 n-1)(2 n+1)$
b. A sequence $\left\{\mathrm{C}_{\mathrm{n}}\right\}$ is defined recursively by $C_{n}=3 C_{n-1}-2 C_{n-2}$ for all $n \geq 3$ with $\mathrm{C}_{1}=5$ and $C_{2}=3$ as initial conditions. Show that $C_{n}=-2^{n}+7$.
c. If $A=\{1,2,3,4\}, B=\{2,5\}, C=\{3,4,7\}$, determine $\mathrm{A} \times \mathrm{B}, \mathrm{B} \times \mathrm{A}, \mathrm{A} \cup(\mathrm{B} \times \mathrm{C}),(\mathrm{A} \cup \mathrm{B}) \times \mathrm{C}$, $(A \times C) \cup(B \times C)$.

## UNIT - IV

4 a. Let $A=\{1,2,3,4\}$ and $R=\{(1,1),(1,2),(2,1),(3,1),(3,3),(1,3)(4,1)(4,4)\}$ be a relation on A. Is R is an equivalence relation?
b. The digraph for a relation on $\mathrm{A}=\{1,2,3,4\}$ is shown in Fig.4(b), i) Verify that $(\mathrm{A}, \mathrm{R})$ is a posen and find its Hesse diagram and ii) Topological set (A, R)

c. For the Posets shown in Fig. 4(c), find;
i) All upper bounds
ii) LUB and GLB of the set B, where B $=\{3,4,5\}$



5 a . Define the following terms with respect to coding theory:
i) Parity Check code
ii) Hamming distance
iii) Group code
iv) Generator matrix
b. What is cyclic group? Explain and hence show that group ( $\mathrm{G},{ }^{*}$ ) whose multiplication table given is cyclic

| $*$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $a$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
| $b$ | $b$ | $c$ | $d$ | $e$ | $f$ | $a$ |
| $c$ | $c$ | $d$ | $e$ | $f$ | $a$ | $b$ |
| $d$ | $d$ | $e$ | $f$ | $a$ | $b$ | $c$ |
| $e$ | $e$ | $f$ | $a$ | $b$ | $c$ | $d$ |
| $f$ | $f$ | $a$ | $b$ | $c$ | $d$ | $e$ |

c. Define subgroup. If $\mathrm{H}, \mathrm{K}$ are subgroup of G , Prove that $\mathrm{H} \cap \mathrm{K}$ is also subgroup.

